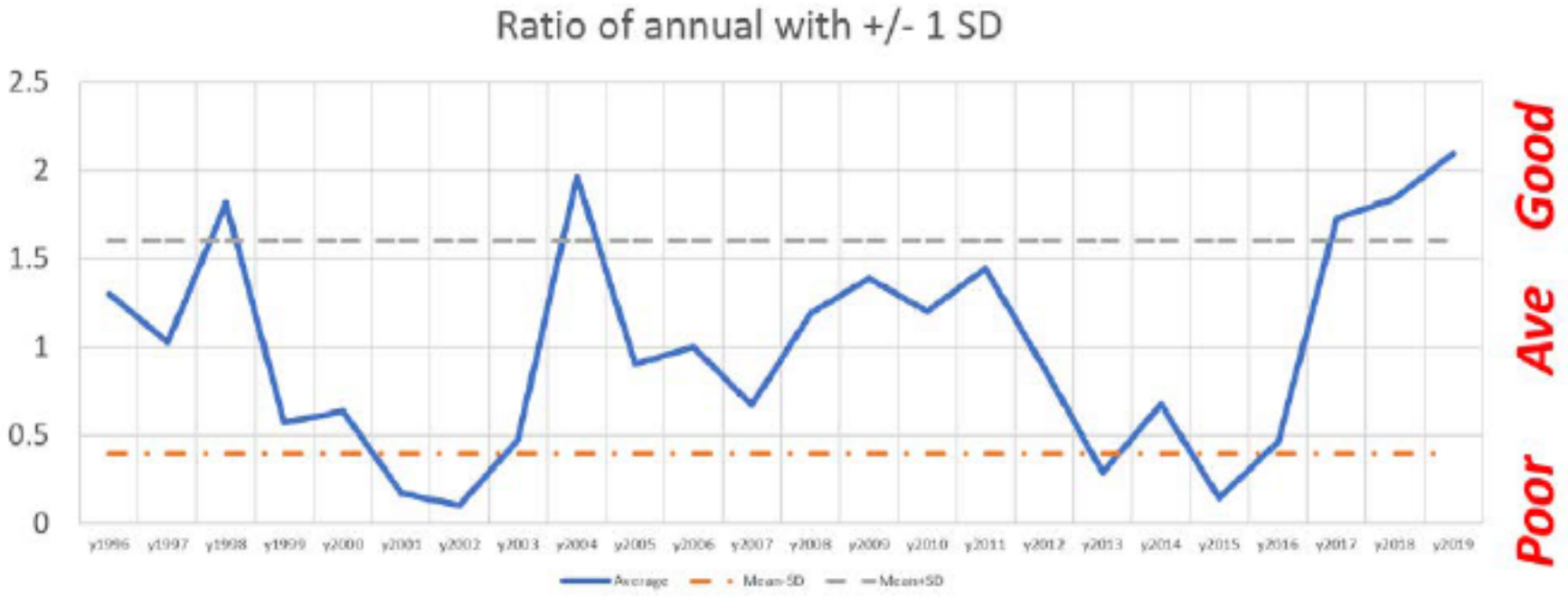
Working Paper #17b
Cusum for Average Weight

In-Season Detection of System State



Good{1998, 2004, 2017, 2018, 2019}

Average {1997, 1999, 2000, 2005-2007, 2010-2012, 2014}

Poor {2001,2002, 2003, 2013, 2015, 2016}

Figure 1. Designation of good, average and poor fishing years based on total landings. The dashed red lines represent +/- 1 SD of the mean. Annual catches were normalized by dividing observations by the overall mean.

← YEAR: 1996-2019 →

The challenge. How to detect pattern early in the year?

← Week of the Year →

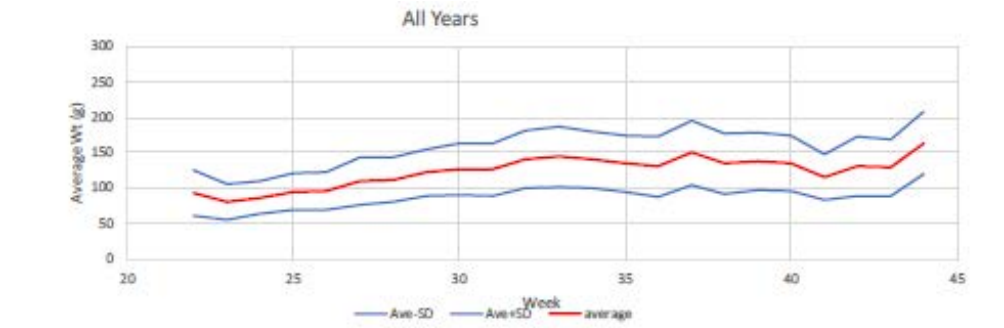
Type	Ave	Ave	Good	Ave	Ave	Poor	Poor	Poor	Good	Ave	Ave	Ave	Ave	Ave	Ave	Ave	Ave	Poor	Ave	Poor	Poor	Good	Good	Good
Type																								
														Fishing Year										
Week	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
21		0.49	0.50						0.83														0.80	0.69
22			0.60						0.79														0.80	0.84
23			0.69	0.58			0.68	0.70	0.85						0.44	0.61	0.59					0.66	0.88	0.52
24		0.48	0.86	0.64		0.61	0.80	0.74	0.91	0.75						0.86	0.58	0.45	0.61			0.70	0.86	
25			0.85	0.73	0.81	0.71	0.74	0.82	0.84	0.67	0.68			0.62	0.72		0.74		0.60			0.82	1.01	1.04
26		0.79	0.84	0.78		0.73	0.86	0.88	0.97	0.74	0.70			0.81	0.95	0.64	0.73	0.50	0.56	0.55	0.63		1.23	0.99
27		0.96	0.92	0.85	0.85	0.72	0.79	0.96	0.99	0.73	0.90				0.87				0.56			0.98	1.37	0.93
28		0.73	1.08	0.84	0.86	0.75	0.82	0.96	1.34	0.77	0.93			0.91	0.97	0.64	0.86	0.52	0.62			1.13	1.51	1.05
29		1.03	1.15	1.23	0.78	0.64	0.96	1.05	1.06	0.88	1.03			1.03	1.01	0.79	1.00			0.68	0.70	1.26	1.44	1.19
30		1.24	1.22	1.11	0.66	0.54	0.92	0.79	1.18	0.91	1.03			1.23	1.04			0.69	0.60			1.54	1.61	1.25
31		1.21	1.29	1.10	0.61		0.95	0.80	1.33	0.95	1.33			1.08		0.82			0.66	0.63	0.64		1.73	1.34
32		1.16	1.28	1.06	0.66		0.85	0.97	1.45	1.01	1.49			1.34	1.04		0.81		0.74	0.75		1.70	1.66	1.43
33		1.38	1.34	0.98			0.95	0.81	1.48	0.98	1.35			1.46		1.06		0.75	0.67		0.54	1.66	1.89	1.46
34		1.34	1.49	1.01		0.73	0.92		1.64	0.98	1.53			1.21	1.01	0.95	0.95		0.80	0.73	0.63	1.75		1.73
35		1.42	1.46	1.15		0.67	0.92	0.88	1.35	0.83	1.60			1.46	1.01	0.88	0.85	0.73	0.77		0.58	2.05		
36		1.47		0.98		0.71	0.71	0.96	1.06	0.80	1.39			1.32	1.02		0.77		0.90	0.69	0.64	1.92		1.64
37		1.60		1.24				0.86	1.08	0.89	1.72			1.23		0.96	0.96	0.75		0.74		1.95		1.90
38		1.33		1.24	1.19		0.98	1.19	1.20	0.93	1.78						0.86		1.01	0.67	0.77			
39		1.47		1.44	1.16			0.99		0.80	1.53			1.13		0.96	0.86	0.81		0.98	0.72			1.69
40		1.46		1.33				1.02		0.84	1.38					0.97	0.92	0.88						
41		1.16					0.95	1.01						1.28		0.99	0.36	0.81						
42		1.59					0.89				1.47					0.95	0.92	0.66		0.66	0.87	1.57		
43		1.47					0.92	1.17			1.27			1.13							0.85		0.56	
44		1.22						1.22								1.18		1.04			0.99		2.32	

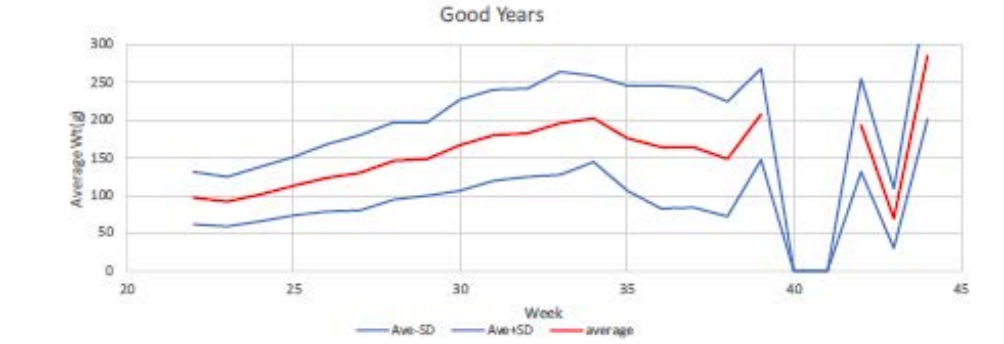
Characterizing the Seasonal pattern of Growth for all years, good years, average years, and poor years.

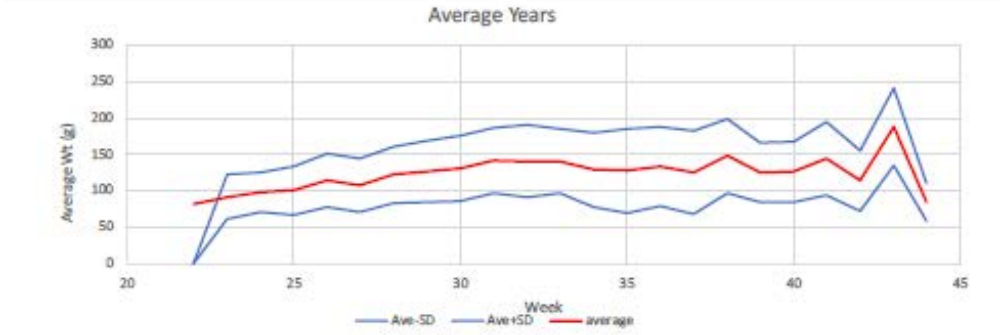
Bounds represent +/- 1 SD of mean.

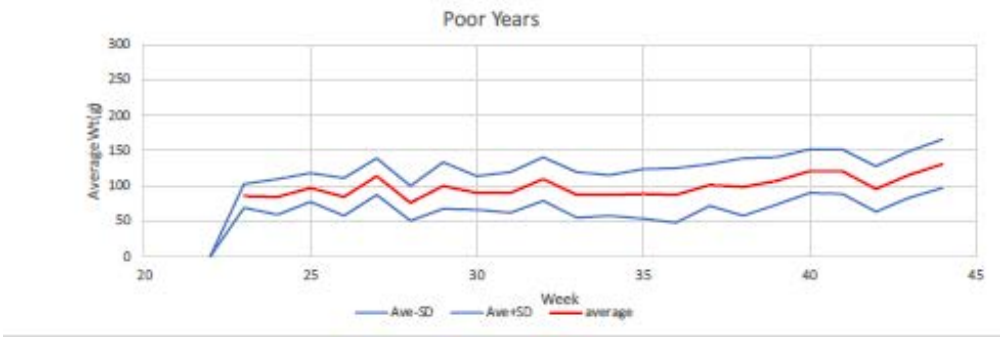
Now—compare each year to these 4 baselines using Cusum method.

Recall that these determinations were based on standardized average landings (Fig. 1) rather than the apparent growth patterns.









Modification when mean and variance of apparent growth *G(w,y)* varies with week (*w*) and year (y):

Let $\mathbf{G}^{w,y}$ equal the average weight observed in week w and year y. Essentially we would like to detect the most likely seasonal distribution of catch rates the $\mathbf{G}^{w,y}$ is drawn from.

$$G(w,y) \sim N(\mu(type,week), \sigma(type,w))$$

Where type= {good, average, poor}

$$C^{+}_{w,y} = \max\left[0, G_{w,y} - \left(\mu_{type,w} + K_{type,w}\right) + C^{+}_{w-1,y)}\right]$$

$$C^{-}_{w,y} = \max\left[0, \left(\mu_{type,w} - K_{type,w}\right) - G_{w,y} + C^{-}_{i-1)}\right]$$

The variance estimates for average weight should be considered a first approximation. Need additional information for weighting of samples by market category.

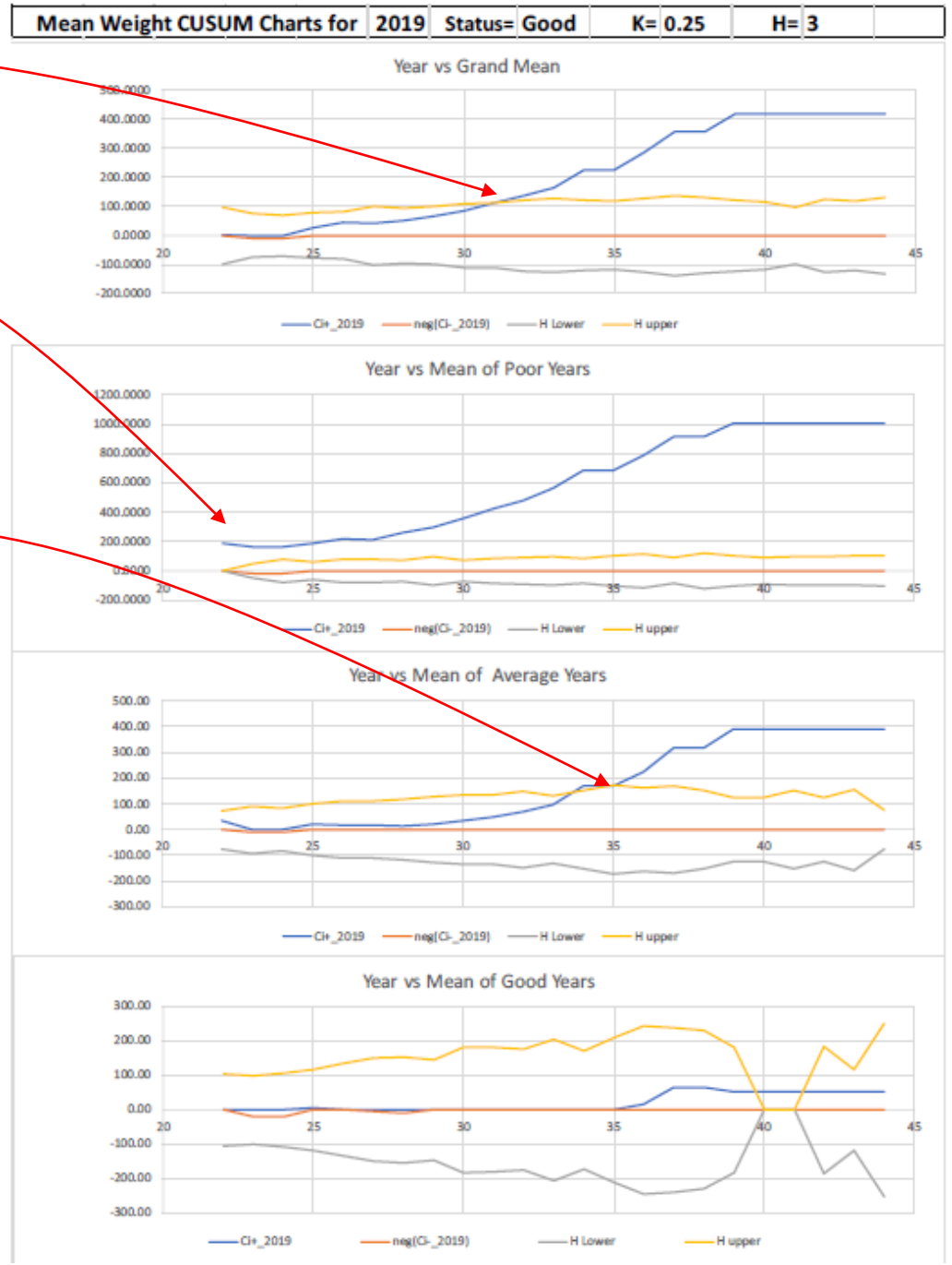
Given these considerations, the buffer $\mathbf{K}^{type,w} = \mathbf{0.25}\ \boldsymbol{\sigma}^{type,w}$ and the upper and lower decision bounds to $\mathbf{H}^{type,w} = \mathbf{3}\ \boldsymbol{\sigma}^{type,w}$. Given an average standard deviation of over all years of ~35 g this modification allows for detection changes apparent growth changes greater than about 8 g with decision bound on the order of 105 g.

Table 1. Summary of Cusum performance for detecting system state (good, average, poor) using slack variable K=0.25 SD and control bounds H=+/- 3SD limits. The response variable is average weight per week. Entries represent the week when the Cusum first exceeded the control limit. The sign following the number represents whether the Cusum statistics exceeded the upper bound (+) or fell below the lower bound (-). Data for 2007 and 2008 were not available when this report was prepared.

		First Out of Bounds Detection Year			
Year	Classification	All Years	Poor Years	Average Years	Good Years
1997	Ave	35+	29+	37+	39+
1998	Good	34+	22+	none	none
1999	Ave	none	29+	none	none
2000	Ave	32-	none	none	31-
2001	Poor	33-	none	34-	28-
2002	Poor	40-	35+	none	32-
2003	Poor	none	37+	none	32-
2004	Good	23+	20+	33+	none
2005	Ave	37-	34+	38-	30-
2006	Ave	34+	30+	36+	40+
2007					
2008					
2009	Ave	41+	29+	none	none
2010	Ave	none	29+	none	33-
2011	Ave	38-	none	38-	32-
2012	Ave	38-	none	38-	33-
2013	Poor	29-	none	33-	28-
2014	Ave	29-	none	29-	27-
2016	Poor	33-	none	33-	32-
2017	Good	32+	28+	33+	none
2018	Good	26+	22+	27+	none
2019	Good	31+	22+	35+	none

Scoring the best year: 2019

		First Out of Bounds Detection Year			
Year	Classification	All Years	Poor Years	Average Years	Good Years
1997	Ave	35+	29+	37+	39+
1998	Good	34+	22+	none	none
1999	Ave	none	29+	none	none
2000	Ave	32-	none	none	31-
2001	Poor	33-	none	34-	28-
2002	Poor	40-	35+	none	32-
2003	Poor	none	37+	none	32-
2004	Good	23+	20+	33+	none
2005	Ave	37-	34+	38-	30-
2006	Ave	34+	30+	36+	40+
2007					
2008					
2009	Ave	41+	29+	none	none
2010	Ave	none	29+	none	33-
2011	Ave	38-	none	38-	32-
2012	Ave	38-	none	38-	33-
2013	Poor	29-	none	33-	28-
2014	Ave	29-	none	29-	27-
2016	Poor	33-	none	33-	32-
2017	Good	32+	28+	33+	none
2018	Good	26+	22+	27+	none
2019	Good	31+	22+	35+	none



Scoring a good year: 2018

		First Out of Bounds Detection Year			
Year	Classification	All Years	Poor Years	Average Years	Good Years
1997	Ave	35+	29+	37+	39+
1998	Good	34+	22+	none	none
1999	Ave	none	29+	none	none
2000	Ave	32-	none	none	31-
2001	Poor	33-	none	34-	28-
2002	Poor	40-	35+	none	32-
2003	Poor	none	37+	none	32-
2004	Good	23+	20+	33+	none
2005	Ave	37-	34+	38-	30-
2006	Ave	34+	30+	36+	40+
2007					
2008					
2009	Ave	41+	29+	none	none
2010	Ave	none	29+	none	33-
2011	Ave	38-	none	38-	32-
2012	Ave	38-	none	38-	33-
2013	Poor	29-	none	33-	28-
2014	Ave	29-	none	29-	27-
2016	Poor	33-	none	33-	32-
2017	Good	32+	28+	33+	none
2018	Good	26+	22+	27+	none
2019	Good	31+	22+	35+	none

Scoring a Poor Year 2001

		First Out of Bounds Detection Year			
Year	Classification	All Years	Poor Years	Average Years	Good Years
1997	Ave	35+	29+	37+	39+
1998	Good	34+	22+	none	none
1999	Ave	none	29+	none	none
2000	Ave	32-	none	none	31-
2001	Poor	33-	none	34-	28-
2002	Poor	40-	35+	none	32-
2003	Poor	none	37+	none	32-
2004	Good	23+	20+	33+	none
2005	Ave	37-	34+	38-	30-
2006	Ave	34+	30+	36+	40+
2007					
2008					
2009	Ave	41+	29+	none	none
2010	Ave	none	29+	none	33-
2011	Ave	38-	none	38-	32-
2012	Ave	38-	none	38-	33-
2013	Poor	29-	none	33-	28-
2014	Ave	29-	none	29-	27-
2016	Poor	33-	none	33-	32-
2017	Good	32+	28+	33+	none
2018	Good	26+	22+	27+	none
2019	Good	31+	22+	35+	none

Scoring an Average Year: 2011

		First Out of Bounds Detection Year			
Year	Classification	All Years	Poor Years	Average Years	Good Years
1997	Ave	35+	29+	37+	39+
1998	Good	34+	22+	none	none
1999	Ave	none	29+	none	none
2000	Ave	32-	none	none	31-
2001	Poor	33-	none	34-	28-
2002	Poor	40-	35+	none	32-
2003	Poor	none	37+	none	32-
2004	Good	23+	20+	33+	none
2005	Ave	37-	34+	38-	30-
2006	Ave	34+	30+	36+	40+
2007					
2008					
2009	Ave	41+	29+	none	none
2010	Ave	none	29+	none	33-
2011	Ave	38-	none	38-	32-
2012	Ave	38-	none	38-	33-
2013	Poor	29-	none	33-	28-
2014	Ave	29-	none	29-	27-
2016	Poor	33-	none	33-	32-
2017	Good	32+	28+	33+	none
2018	Good	26+	22+	27+	none
2019	Good	31+	22+	35+	none

Scoring a Poor Year 2013

		First Out of Bounds Detection Year			
Year	Classification	All Years	Poor Years	Average Years	Good Years
1997	Ave	35+	29+	37+	39+
1998	Good	34+	22+	none	none
1999	Ave	none	29+	none	none
2000	Ave	32-	none	none	31-
2001	Poor	33-	none	34-	28-
2002	Poor	40-	35+	none	32-
2003	Poor	none	37+	none	32-
2004	Good	23+	20+	33+	none
2005	Ave	37-	34+	38-	30-
2006	Ave	34+	30+	36+	40+
2007					
2008					
2009	Ave	41+	29+	none	none
2010	Ave	none	29+	none	33-
2011	Ave	38-	none	38-	32-
2012	Ave	38-	none	38-	33-
2013	Poor	29-	none	33-	28-
2014	Ave	29-	none	29-	27-
2016	Poor	33-	none	33-	32-
2017	Good	32+	28+	33+	none
2018	Good	26+	22+	27+	none
2019	Good	31+	22+	35+	none

Making Real-Time Decisions

- In an ideal system, one wants to detect the condition {good, average, bad} as early as possible, leaving more time for making a decision and implementing the regulatory process.
- Need to have a drop dead date for decision. If insufficient signal at drop dead date, then no quota adjustment.
- Need to control for false positives (quota increased with insufficient evidence) and false negatives (quota unchanged but evidence is sufficient)
- All of the above discussion assumes that the initial classification of year status is appropriate
  - Solicitation of input from harvesters and processors is essential for this determination.
  - Other candidate metrics for system identification could also be included

Example Decision Table

		Week when Cusum Statistic Exceeds H bound				
	Test Year	*Poor*	*Average*	*Good*	Determi-nation	Decision
Ideal Detection Signals	Y1	0 or T_1^-	T_2^-	T_3^-	Poor	Same Q
	Y2	T_4^+	0 or T_5^-	T_6^-	Average	Same Q
	Y3	T_7^+	T_8^+	0 or T_9^+	Good	Increase Q at min $\{T_8, T_9\}$
Indeterminate	Y4	T_{10}^+	0	0	Not Good	Same Q
	Y5	0	0	0	Not Good	Same Q

Landings Cusum Results (K=1σ, H=5σ)

		First Out of Bounds Detection Year			
Year	Classification	All Years	Poor Years	Average Years	Good Years
1996	Ave	43+	20+	44+	28-
1997	Ave	none	27+	none	24-
1998	Good	30+	20+	25+	none
1999	Ave	none	30+	none	27-
2000	Ave	none	36+	none	27-
2001	Poor	none	none	none	24-
2002	Poor	none	none	none	24-
2003	Poor	none	43+	none	25-
2004	Good	38+	21+	28+	39+
2005	Ave	none	20+	none	28-
2006	Ave	none	27+	none	28-
2007	Ave	none	35+	none	24-
2008	Ave	42+	31+	45+	24-
2009	Ave	none	25+	none	28-
2010	Ave	none	21+	none	none
2011	Ave	none	20+	26+	39+
2012	Ave	none	33+	none	27-
2013	Poor	none	none	none	24-
2014	Ave	none	33+	none	27-
2015	Poor	none	none	none	24-
2016	Poor	45+	none	none	24-
2017	Good	38+	22+	32+	none
2018	Good	29+	21+	27+	none
2019	Good	31+	21+	27+	none

Average Weight Cusum Results K=0.25σ, H=3σ

		First Out of Bounds Detection Year			
Year	Classification	All Years	Poor Years	Average Years	Good Years
1997	Ave	35+	29+	37+	39+
1998	Good	34+	22+	none	none
1999	Ave	none	29+	none	none
2000	Ave	32-	none	none	31-
2001	Poor	33-	none	34-	28-
2002	Poor	40-	35+	none	32-
2003	Poor	none	37+	none	32-
2004	Good	23+	20+	33+	none
2005	Ave	37-	34+	38-	30-
2006	Ave	34+	30+	36+	40+
2007					
2008					
2009	Ave	41+	29+	none	none
2010	Ave	none	29+	none	33-
2011	Ave	38-	none	38-	32-
2012	Ave	38-	none	38-	33-
2013	Poor	29-	none	33-	28-
2014	Ave	29-	none	29-	27-
2016	Poor	33-	none	33-	32-
2017	Good	32+	28+	33+	none
2018	Good	26+	22+	27+	none
2019	Good	31+	22+	35+	none

Earliest Decision Week Range for "better than average" condition

1998= {25, ?}

2004= {28, 33}

2017= {32,33}
2018= {27, 27}
2019= {27,35}

Sources of Uncertainty

- Basis for original classification of “Good”, “Average”, “Poor”
- Specification of seasonal means and variances based on samples
- Setting the control limits for a time varying mean and variance. Simulations may be useful.
- Better estimate of variance of average weights