

# Part 1: Application of Cusum Method for In-Season Detection

## of Fishery Condition for Illex Squid; Landings, 1996-2019

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### INTRODUCTION

In the absence of an accepted stock assessment model for evaluating the effects of fishing on the Illex resource, management relies on monitoring a quota based on a surplus production model from the 1996 (NEFSC 1996) assessment. The quota has been adjusted slightly upwards several times since then. Under current levels of fishing effort, the quota has not been limiting except for five years since 1996 (i.e., 1998, 2004, 2017, 2018 and 2019). Fisherman noted high catch rates at the end of the season for three consecutive years, suggesting a possible change in either abundance or availability in the Northeast continental shelf waters. It is reasonable to question whether a higher quota might be allowable under such circumstances. If a quota adjustment was an admissible policy, the mechanics of actually implementing it come to the forefront. This working paper addresses the first step of implementation—detecting conditions in terms of landings rate that may warrant a change. A companion working paper examines the utility of changes in average weight per squid as another indicator.

### DATA

Weekly landings data for Illex squid for 1996 to 2019 were provided by Jason Didden. Data consisted of records of total landing by standardized week. These are the same data used by the GARFO for quota monitoring. I also thank Lisa Hendrickson, NEFSC for providing me with the final VTR adjusted data with sub trip identification for 1996-2018.

### METHODS

Cumulative sum (or Cusum) methods are widely used in statistical process control where real-time decision-making is important for maintaining quality standards for a product or a process. The basic idea is to collect information that appropriately identifies when the process is out of control and avoids erroneous identification of out-of-control processes. Letting a process continue when the quality has degraded is bad for future revenue (i.e., defective widgets). Conversely, incorrectly stopping a production process to fix something that is working fine, increases costs and reduces profits.

The Cusum method was first proposed by Page (1954, 1961) and has been refined by many authors since then. The basic concept is simple. Consider a sequence of random variables  $x_i$  drawn from a normal distribution with mean  $\mu_o$  and variance  $\sigma^2$ . If  $\mu_o$  is known, then the expected value of  $\Sigma(x_i - \mu_o) = 0$ . If the mean changes to a new value  $\mu_n$ , but  $\mu_o$  is assumed, then  $\Sigma(x_i - \mu_o)$  will increase with the number observations. Conversely, if the new mean is less than the hypothesized mean, then the cumulative sum will become increasingly negative. Cusum methods take advantage of this expected change to define upper and lower control limits sufficient to detect such changes in real time. Among the advantages of Cusum is earlier detection compared to other methods such as Shewhart control charts.

The methodology herein is based on the so-called tabular method of Montgomery (1996) but modified to account for a seasonally varying mean and variance.

First let's consider a constant mean and variance. The upper and lower cusums are denoted as  $C_i^+$  and  $C_i^-$  and defined by the following recursive equations:

$$C_i^+ = \max [0, x_i - (\mu_0 + K) + C_{i-1}^+ ]$$

$$C_i^- = \max [0, (\mu_0 - K) - x_i + C_{i-1}^- ]$$

Starting values for  $C_i^+$  and  $C_i^-$  are both set to zero for  $i=1$ . The parameter  $\mathbf{K}$  is called the “slack” variable as it acts like a buffer or tolerance level. Changes in  $\mathbf{x}_i$  of less than  $\mathbf{K}$  are essentially zeroed out. For example  $C_{i+1}^+ > C_i^+$  increases only when  $\mathbf{x}_i > \mathbf{u}_0 + \mathbf{K}$  and  $C_{i+1}^- < C_i^-$  only when  $\mathbf{x}_i < \mathbf{u}_0 - \mathbf{K}$ .

$\mathbf{K}$  is generally expressed as a function of the standard deviation. The parameter  $\mathbf{K}$ , written as  $\mathbf{K} = \delta\sigma$  represents the magnitude of the change one wishes to detect in  $\mu$ . The expected number of samples necessary to detect this change is called the average run length (ARL). In other words, ARL is the average number of samples required before determination of true out of control state is determined given an acceptable level of “slack” in the underlying process.

The ARL statistic is determined as a function of  $\mathbf{K}$  and another parameter called  $\mathbf{H}$  where  $\mathbf{H} = \gamma\sigma$ . Montgomery recommends letting  $\gamma$  equal 4 or 5 in order to detect changes within reasonably small number of samples (ARL). For  $\delta=1$  and  $\gamma=5$  the ARL is 10.4; if  $\gamma=4$  then ARL is 8.38. In general terms ARL decreases as  $\delta$  and  $\gamma$  increase. The process is judged to be out of control when  $C_i^+ > \mathbf{H}$  or  $C_i^- < -\mathbf{H}$ .

The above description of Cusum assumes that the mean  $\mu$  and variance  $\sigma^2$  are static over time. For our application we are interested in determining whether we can detect a change in an underlying mean that changes seasonally. Total landings were used to assign 3 categories of fishery system status. In the first category “Good” were annual totals that were more than one standard deviation above the long term mean. The “Poor” category was defined as total catches below one standard deviation and “Average” was defined as everything else (Figure 1). This method identifies the 5 years in which the quota was restrictive, and there was general agreement among fishermen about the fishing conditions in the “poor” years. It is recognized that many factors can influence fishing success including market prices and availability of alternative species. As a first approximation this simple approach appears reasonable.

If the dynamics of fishing success were constant over seasons, then it would be much simpler to diagnose the system state early in the year. Figure 2 illustrates that the attainment of the total seasonal catch not only occurs at different times but also at different rates. Poor years are characterized by phlegmatic rates of change whereas good years increase rapidly between weeks 25 and 35. Interestingly, good and average years tend to be indistinguishable until about week 30. Figure 2.5 depicts the standardized weekly catch rates for each year. Data were standardized by dividing each observation by the average weekly catch over the 1996-2019 period. The good years are readily identified but timing and duration of fishing activity varies greatly over years. (I thank Ben Galuardi, GARFO for providing a similar color coded graphic to show seasonal changes.) The Cusum methodology was tweaked to help identify these qualitative perceptions more quantitatively.

Let  $L_{w,y}$  equal the landings observed in week  $w$  and year  $y$ . Essentially we would like to know what is the most likely seasonal distribution of catch rates the  $L_{w,y}$  is drawn from. Let  $\mu_{all,w}$ ,  $\mu_{good,w}$ ,  $\mu_{ave,w}$ , and  $\mu_{poor,w}$  represent the seasonal means for all samples, the good years, the average years and the poor years, respectively. For each week and type, it is possible to compute a standard deviation as well (i.e.,  $\sigma_{all,w}$ ,  $\sigma_{good,w}$ ,  $\sigma_{ave,w}$ , and  $\sigma_{poor,w}$  ) These values are used to create Figure 3. The Cusum statistics  $C_i^+$  and  $C_i^-$  can be redefined as

$$C_{w,y}^+ = \max [0, L_{w,y} - (\mu_{type,w} + K_{type,w}) + C_{w-1,y}^+ ]$$

$$C_{w,y}^- = \max [0, (\mu_{type,w} - K_{type,w}) - L_{w,y} + C_{w-1,y}^- ]$$

Where  $K_{type,w} = k \sigma_{type,w}$  where  $k$  is a constant=1.  $H$  is also redefined as  $H_{type,w} = h \sigma_{type,w}$  where  $h$  is a constant=5. Thus the Cusum process is specified to detect changes of 1 SD and is declared “out of control” when  $C_{w,y}^+ > H_{type,w}$  or when  $C_{w,y}^- < -H_{type,w}$ . In other words, if the Cusum statistics lie outside the  $H$  bounds, then one would reject the hypothesis that year in question was from a given type. Thus one would expect a putative good year, say 2018 to be rejected as an average or poor year by having a cusum statistic  $C_{w,2018}^+$  above  $H_{poor,w}$ , or  $H_{ave,w}$  for some week  $w$ . Similarly, a putative poor year should have  $C_{w,2016}^- < -H_{good,w}$  and  $C_{w,2016}^- < -H_{ave,w}$ . The test statistics  $C$  for 2018 and 2016 should be close the upper and lower  $H$  boundaries, respectively for the  $H_{all,w}$  comparison.

## RESULTS and DISCUSSION

Seasonal landings data for each year were compared to the seasonal means and standard deviations derived from using all the data, the good years only, the average years only, and the poor years only (Figures 4 to 27). The visualizations are important because they allow for finer scale analyses of pattern. For example, in some years the  $C$  statistics exceeded the upper out of bounds limits for one week only and then returned to a lower value. In other instances the  $C$  statistic closely tracts the upper or lower bound; if the  $H$  limits had been set lower, say  $4\sigma$  rather than  $5\sigma$ , the process would have been declared out of control.

Table 1 provides a concise summary of the statistical results. Results can be scored by comparing the a priori designation of system state to the Cusum determination. Ideally one would like to see the Cusum sequence to determine that the candidate year was “in control” when compared to the baseline years that determined to the results when the process is determined to be out of control. Cusum performance for putative good years {1998, 2004, 2017, 2018, 2019} was as expected. All of the good years, when compared to the poor year set {2001,2002, 2003, 2013, 2015, 2016} were judged out of control before week 22. Comparisons of good years with the average years {1996, 1997, 1999, 2000, 2005-2012, 2014} were judged out of control as early as week 25 and as late as week 32 (Table 1). Hence it would seem that a future “good” year could be identified as above average by the end of July.

It is recognized that the year comparisons to the other years with equal system state are not out of sample comparisons and therefore have some bias. Thus 2019 is compared to the set of years designated as “good” {1998, 2004, 2017, 2018, 2019} rather than a true out of sample set where {2019} is compared to {1998, 2004, 2017, 2018}. Comparisons of each year with all years revealed either all the  $C$  statistics within the  $H$  bounds (identified as “none” in Table 1) or the

out of bounds determination occurred largely after the fishery had closed. Thus 1998, 2018 and 2019 were identified as different from the overall baseline seasonal pattern after the quota had closed the fishery. Poor years were generally distinguishable from good years and performed as expected with a median week determination of 24.5 weeks. Thus poor years could be distinguished from good years by the end of June. Average years were distinguished as being lower than good years in 11 of the 13 years. Curiously, 2010 and 2011 were essentially indistinguishable from the good year pattern until about week 39. The timing of such a determination would not be relevant to management decision making. In an ideal system, one wants to detect the condition {good, average, bad} as early as possible, leaving more time for making a decision and implementing the regulatory process.

The choice of bounds for the slack variable and control variables are matters of policy or risk preference. When  $K=1\sigma$  one is stating that a process will be declared out of control when the new mean drifts by one standard deviation (i.e.,  $\mu_n=\mu_0+\sigma$ ). As  $K$  increases, one is essentially saying that changes in the underlying mean of differences less than  $K$  are acceptable. If  $K=3\sigma$  then an undetected change of  $3\sigma$  is judged acceptable. If the  $C$  exceeds  $H$  for this level of  $K$  then it would only take an average of 2.19 samples (ARL=2.19) to detect this change. For finer levels of control, say  $K=1\sigma$ , the ARL =8.38 when  $H=4\sigma$  and 10.4 when  $K=5\sigma$  (Montgomery 1996, p 322). For the quota monitoring problem under consideration there are no existing guidelines. Montgomery suggests that  $K=0.5\sigma$  and  $H=4\sigma$  or  $5\sigma$  seems to work well in practical studies. The ARL statistics are based on a single change in the mean based on a transition to  $\mu_n$  from  $\mu_0$ . Simulation studies may be necessary to validate the ARL statistics associated with control problem addressed in this paper.

## REFERENCES

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Table 1. Summary of Cusum performance for detecting system state (good, average, poor) using slack variable  $K=1$  SD and control bounds  $H=\pm 5$  SD limits. Entries represent the week when the Cusum first exceeded the control limit. The sign represents whether the Cusum statistics exceeded the upper bound (+) or fell below the lower bound (-).

<i>Year</i>	<i>Classification</i>	<i>First Out of Bounds Detection Year</i>			
		<i>All Years</i>	<i>Poor Years</i>	<i>Average Years</i>	<i>Good Years</i>
1996	Ave	43+	20+	44+	28-
1997	Ave	none	27+	none	24-
1998	Good	30+	20+	25+	none
1999	Ave	none	30+	none	27-
2000	Ave	none	36+	none	27-
2001	Poor	none	none	none	24-
2002	Poor	none	none	none	24-
2003	Poor	none	43+	none	25-
2004	Good	38+	21+	28+	39+
2005	Ave	none	20+	none	28-
2006	Ave	none	27+	none	28-
2007	Ave	none	35+	none	24-
2008	Ave	42+	31+	45+	24-
2009	Ave	none	25+	none	28-
2010	Ave	none	21+	none	none
2011	Ave	none	20+	26+	39+
2012	Ave	none	33+	none	27-
2013	Poor	none	none	none	24-
2014	Ave	none	33+	none	27-
2015	Poor	none	none	none	24-
2016	Poor	45+	none	none	24-
2017	Good	38+	22+	32+	none
2018	Good	29+	21+	27+	none
2019	Good	31+	21+	27+	none

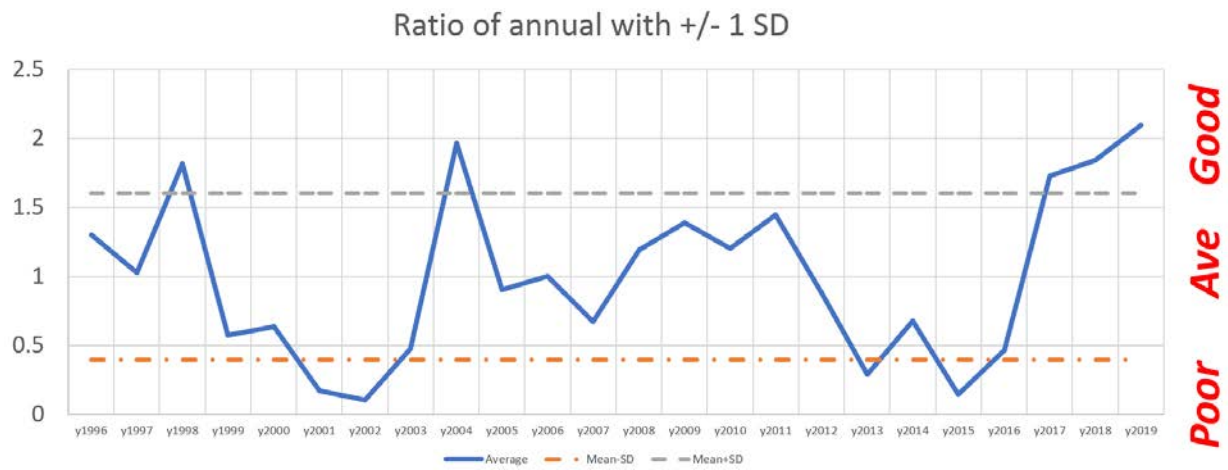


Figure 1. Designation of good, average and poor fishing years based on total landings. The dashed red lines represent +/- 1 SD of the mean. Annual catches were normalized by dividing observations by the overall mean.

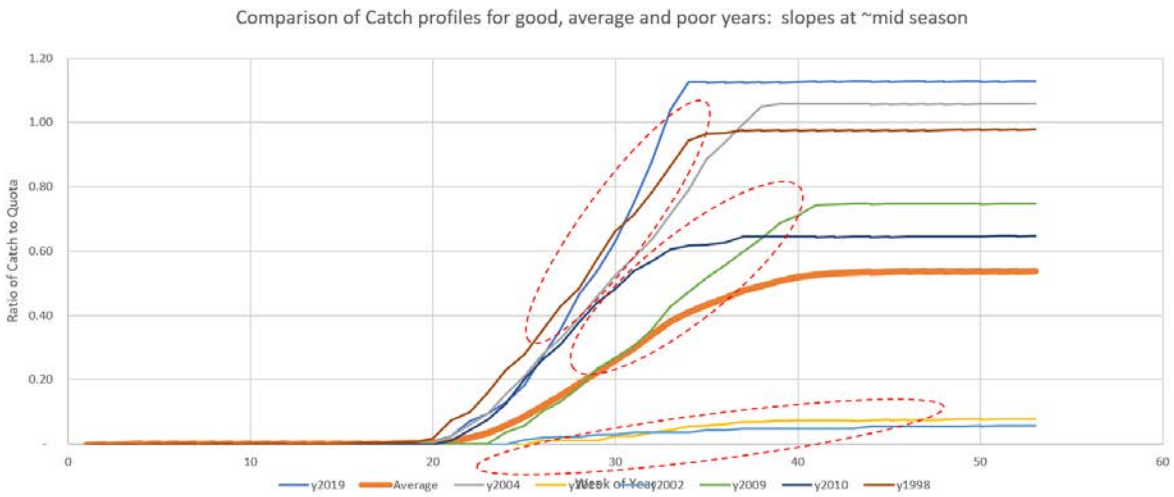
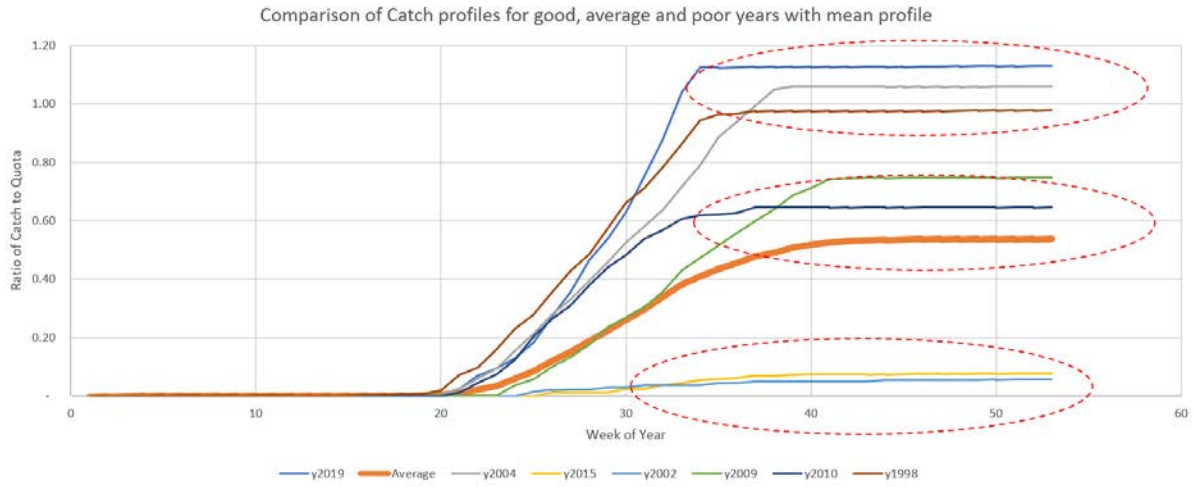
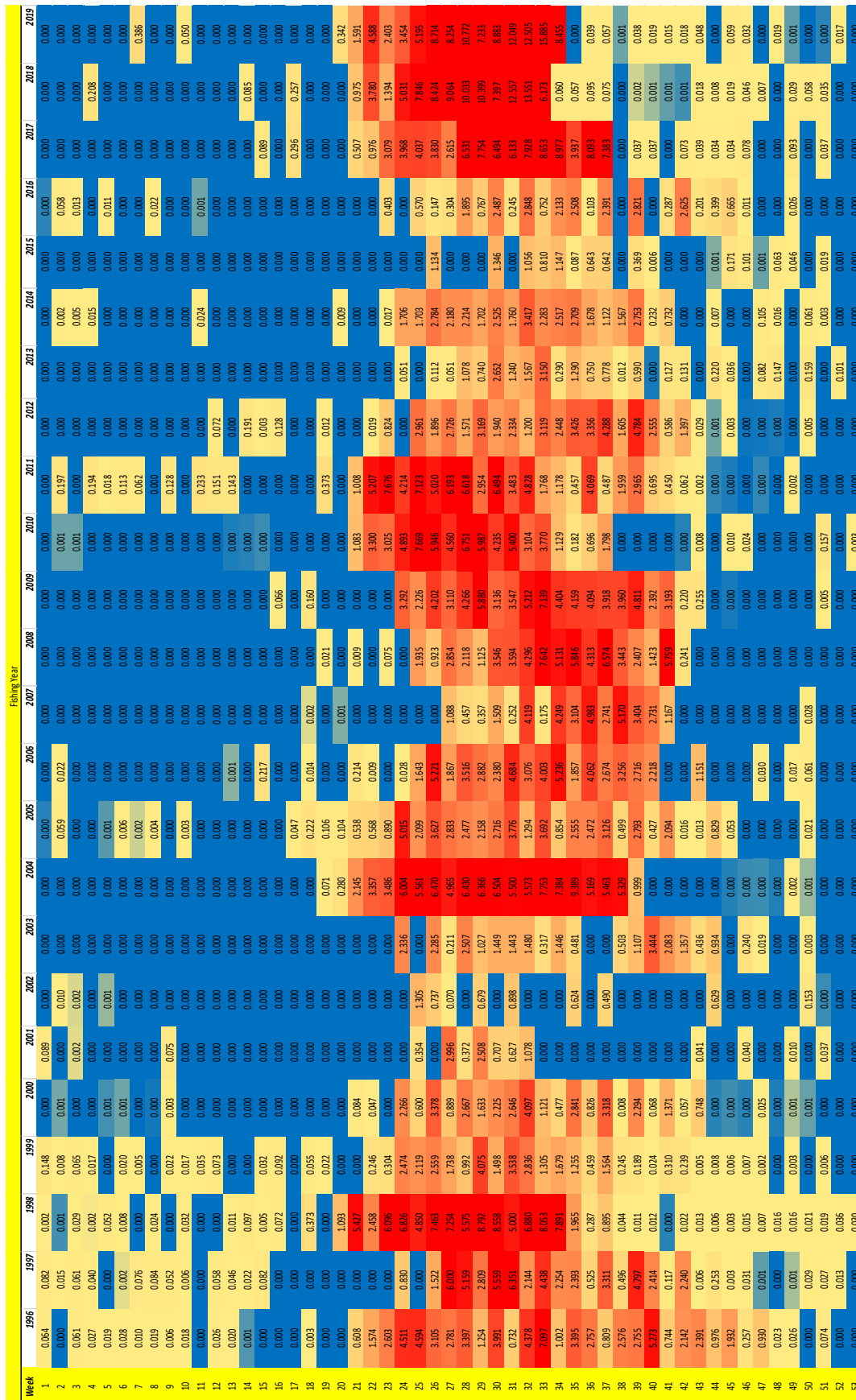


Figure 2. Comparison of seasonal catch patterns for good, average, and poor years to the overall mean pattern (solid thick line). Ellipses illustrate the clustering of patterns by magnitude (top) and rate of increase (bottom) for two example years in the good, average and poor categories.





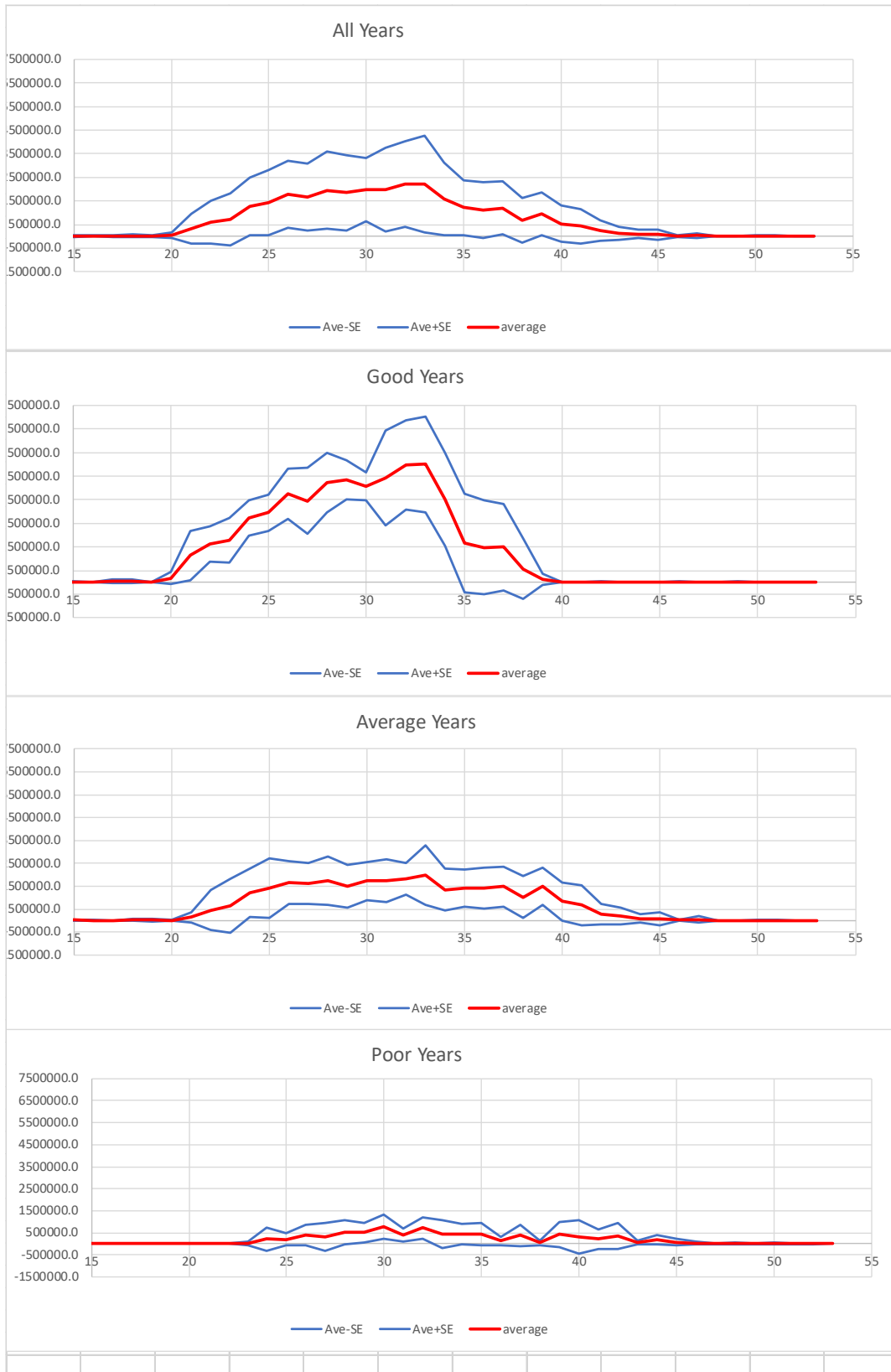


Figure 3. Seasonal catch patterns for all years (top row), good years (row 2), average years (row 3) and poor years (bottom row) expressed in terms of pounds per week.



Figure 4. Cusum statistics for 1996 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 1996 was classified a priori as an average year.





Figure 6. Cusum statistics for 1998 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 1998 was classified a priori as a good year.



Figure 7. Cusum statistics for 1999 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 1999 was classified a priori as an average year.



Figure 8. Cusum statistics for 2000 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2000 was classified a priori as an average year.



Figure 9. Cusum statistics for 2001 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2001 was classified a priori as a poor year.



Figure 10. Cusum statistics for 2002 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2002 was classified a priori as a poor year.





Figure 11. Cusum statistics for 2003 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2003 was classified a priori as a poor year.



Figure 12. Cusum statistics for 2004 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2004 was classified a priori as a good year.



Figure 13. Cusum statistics for 2005 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2005 was classified a priori as an average year.

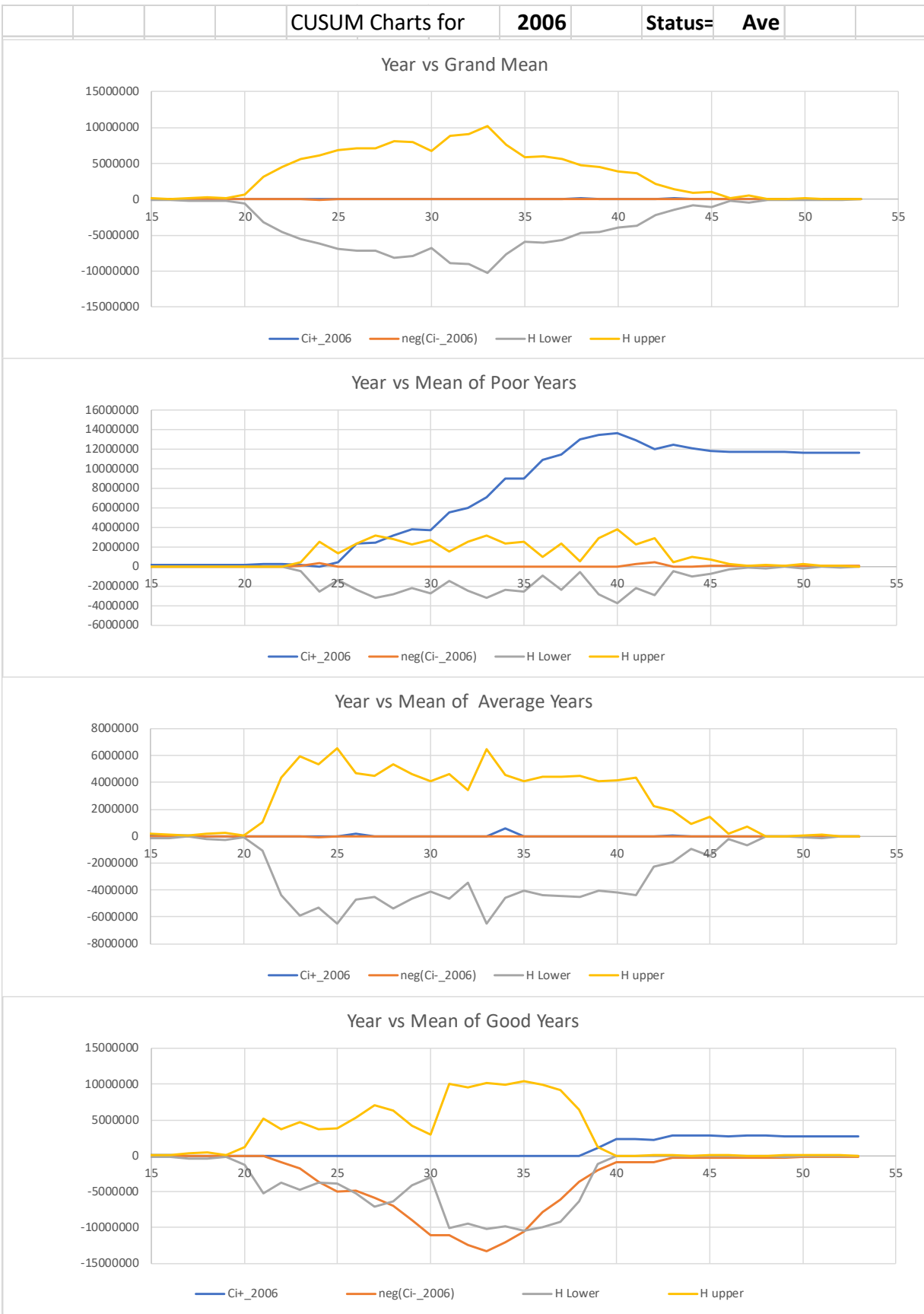


Figure 14. Cusum statistics for 2006 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2006 was classified a priori as an average year.



Figure 15. Cusum statistics for 2007 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary.  $C^+$  is shown in blue;  $C^-$  is shown in red. Based on landings, 2007 was classified a priori as an average year.

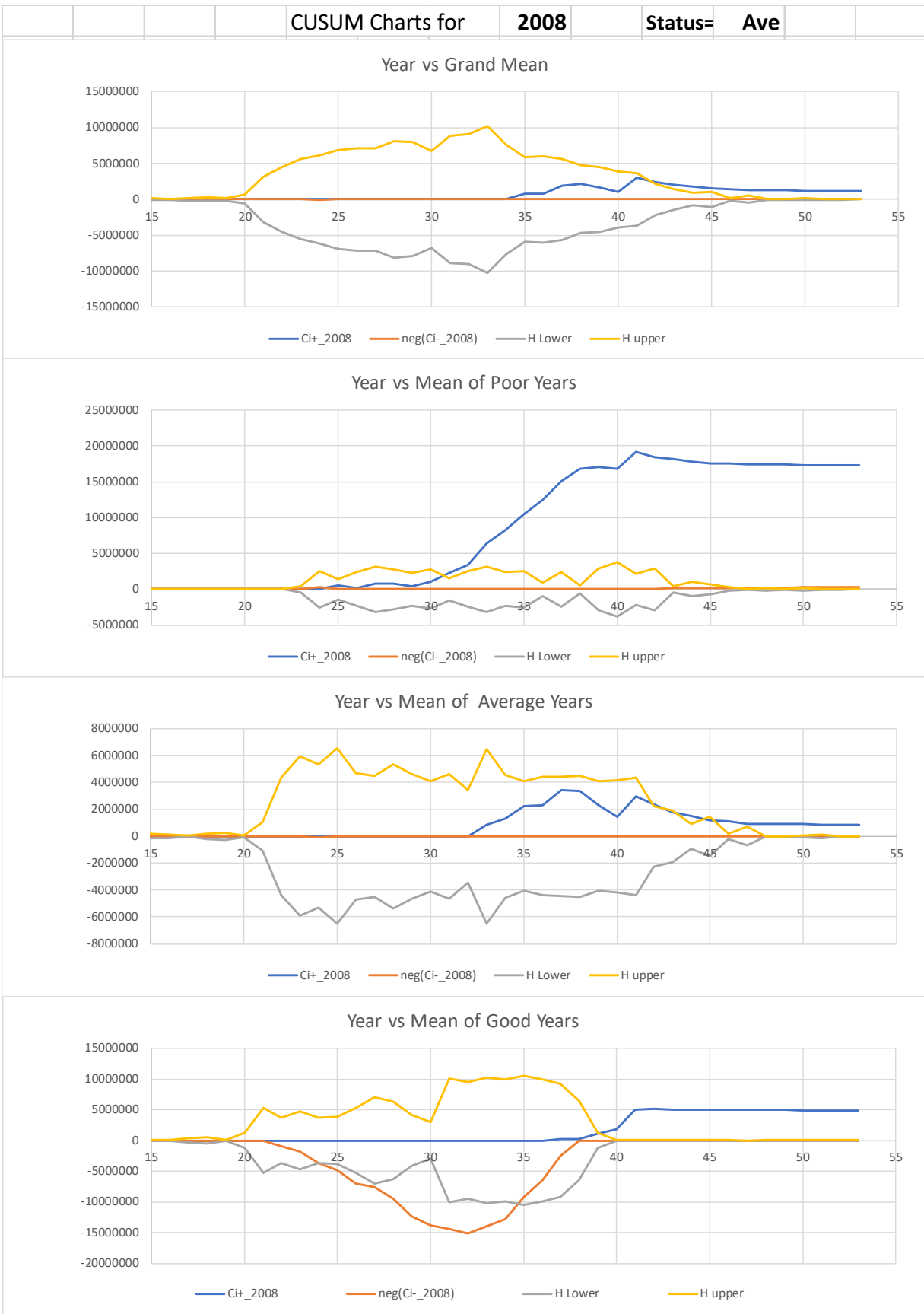


Figure 16. Cusum statistics for 2008 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2008 was classified a priori as an average year.

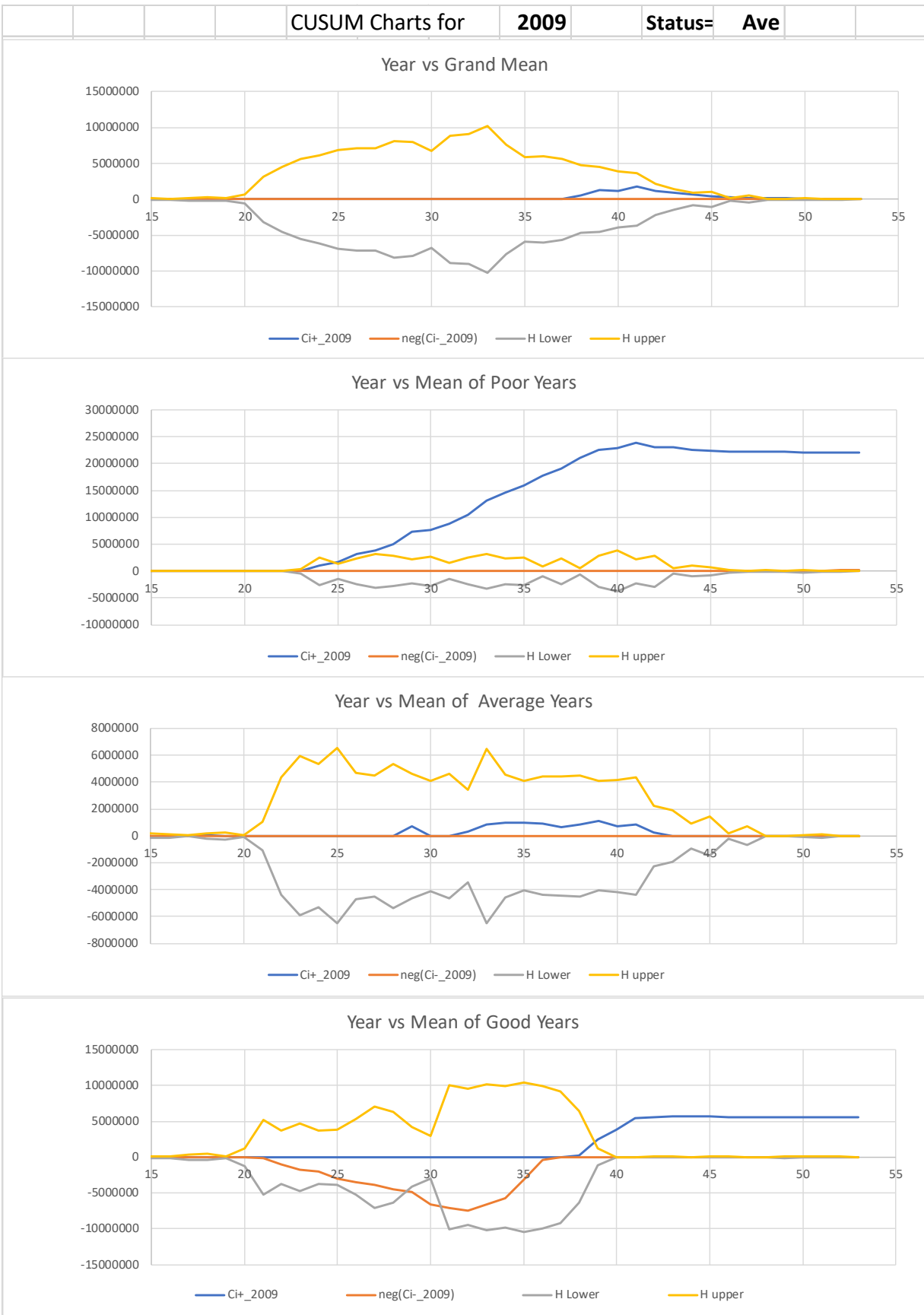


Figure 17. Cusum statistics for 2009 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2009 was classified a priori as an average year.

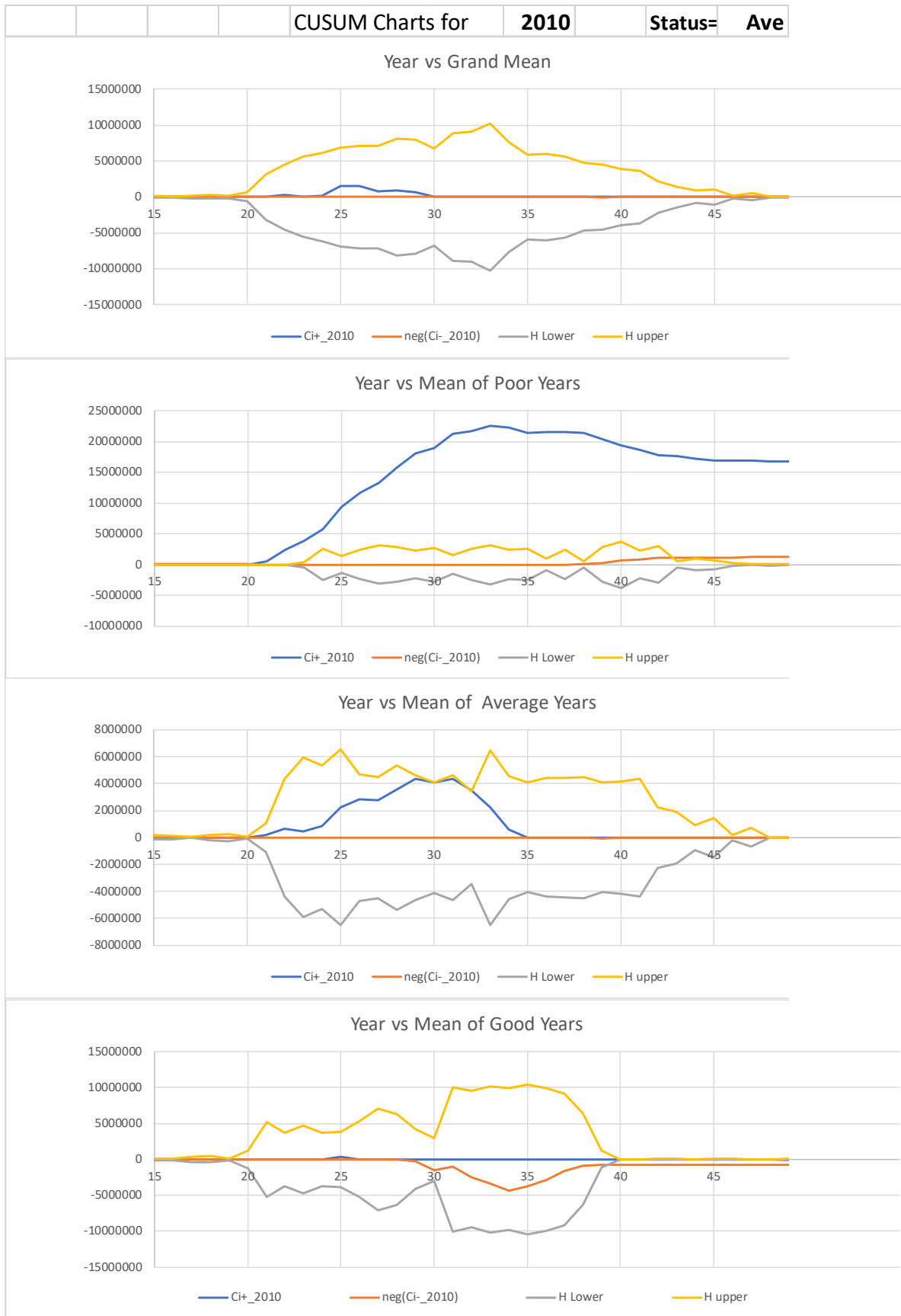


Figure 18. Cusum statistics for 2010 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2010 was classified a priori as an average year.



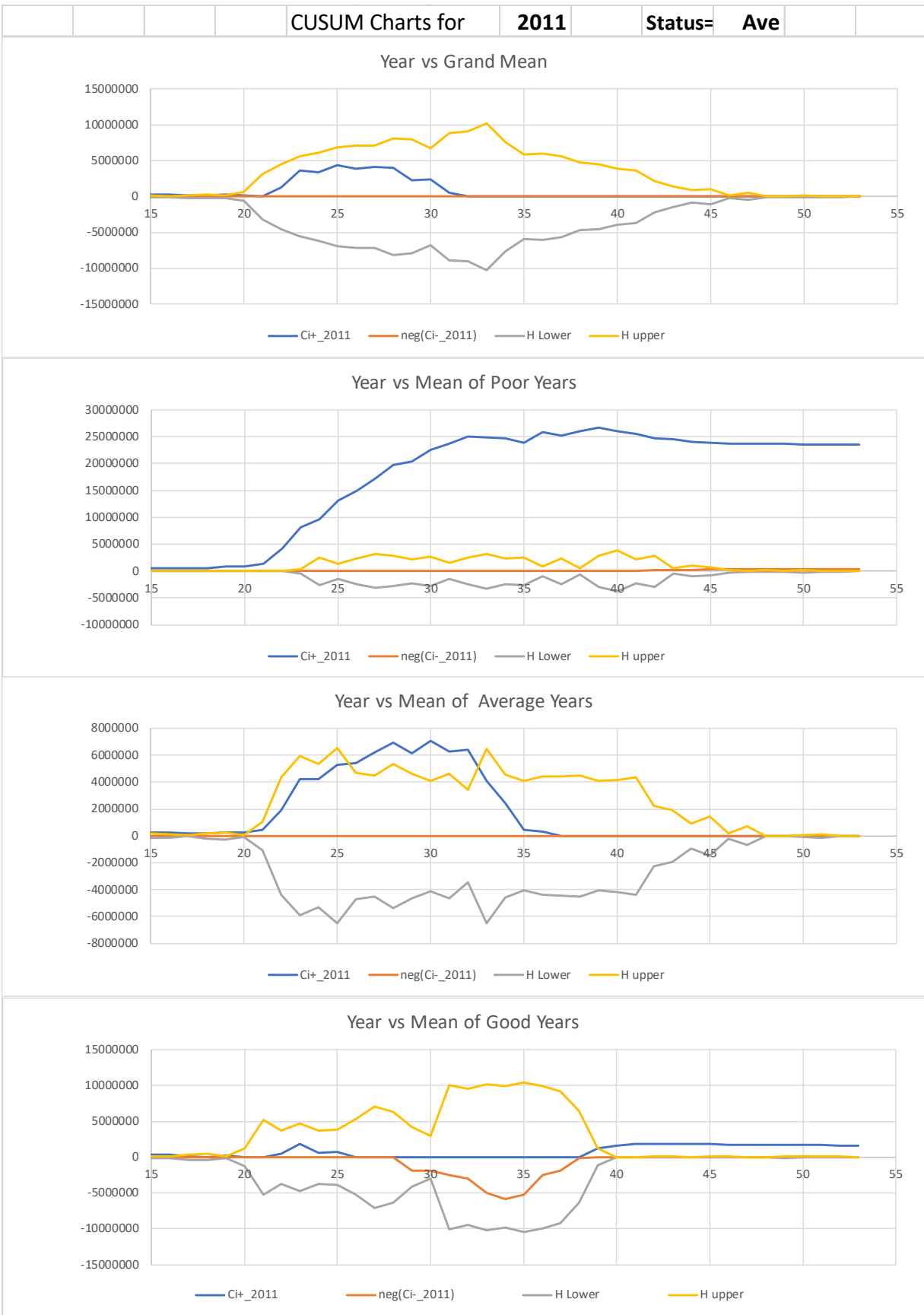


Figure 19. Cusum statistics for 2011 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2011 was classified a priori as an average year.

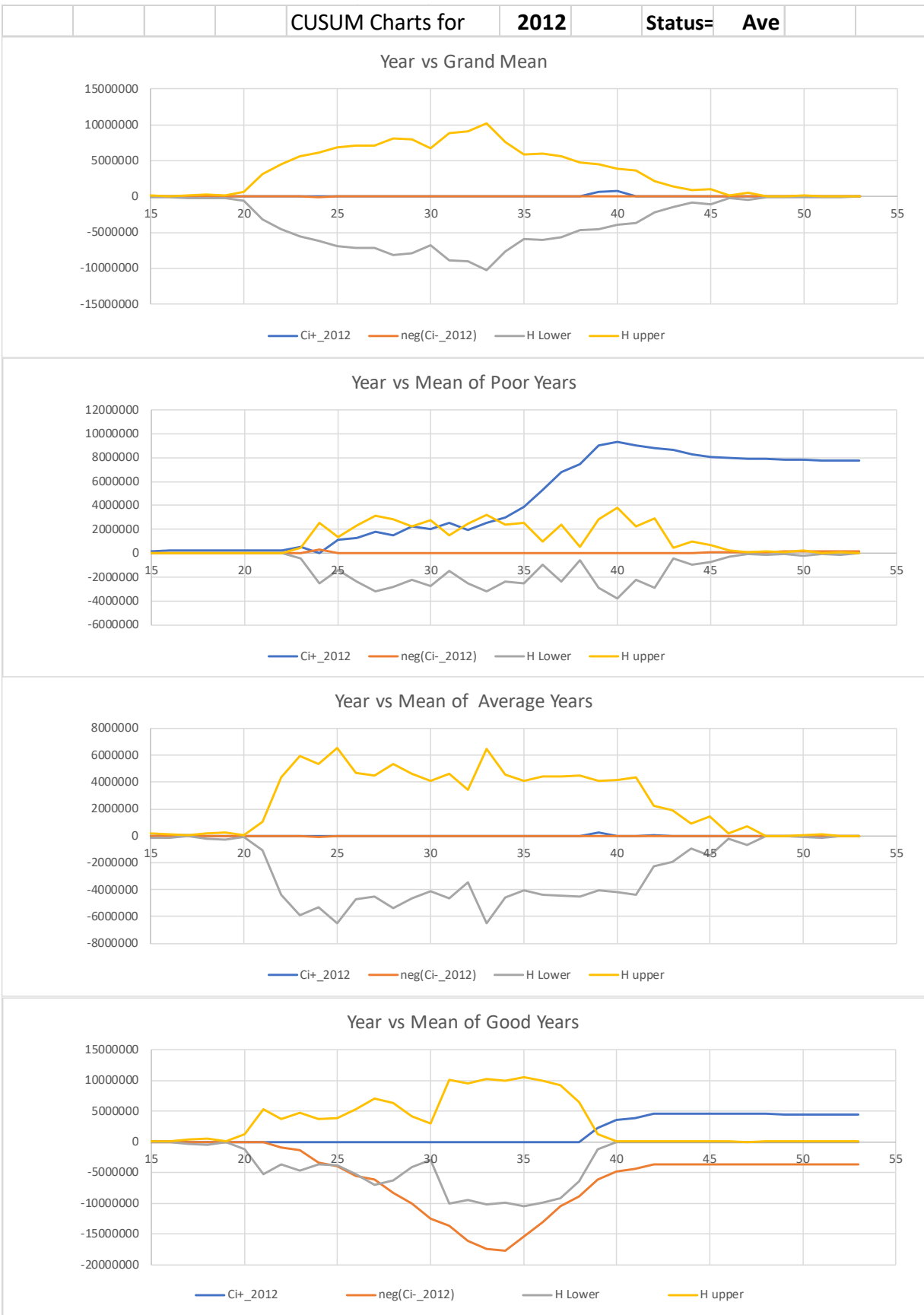


Figure 20. Cusum statistics for 2012 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2012 was classified a priori as an average year.



Figure 21. Cusum statistics for 2013 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2013 was classified a priori as a poor year.

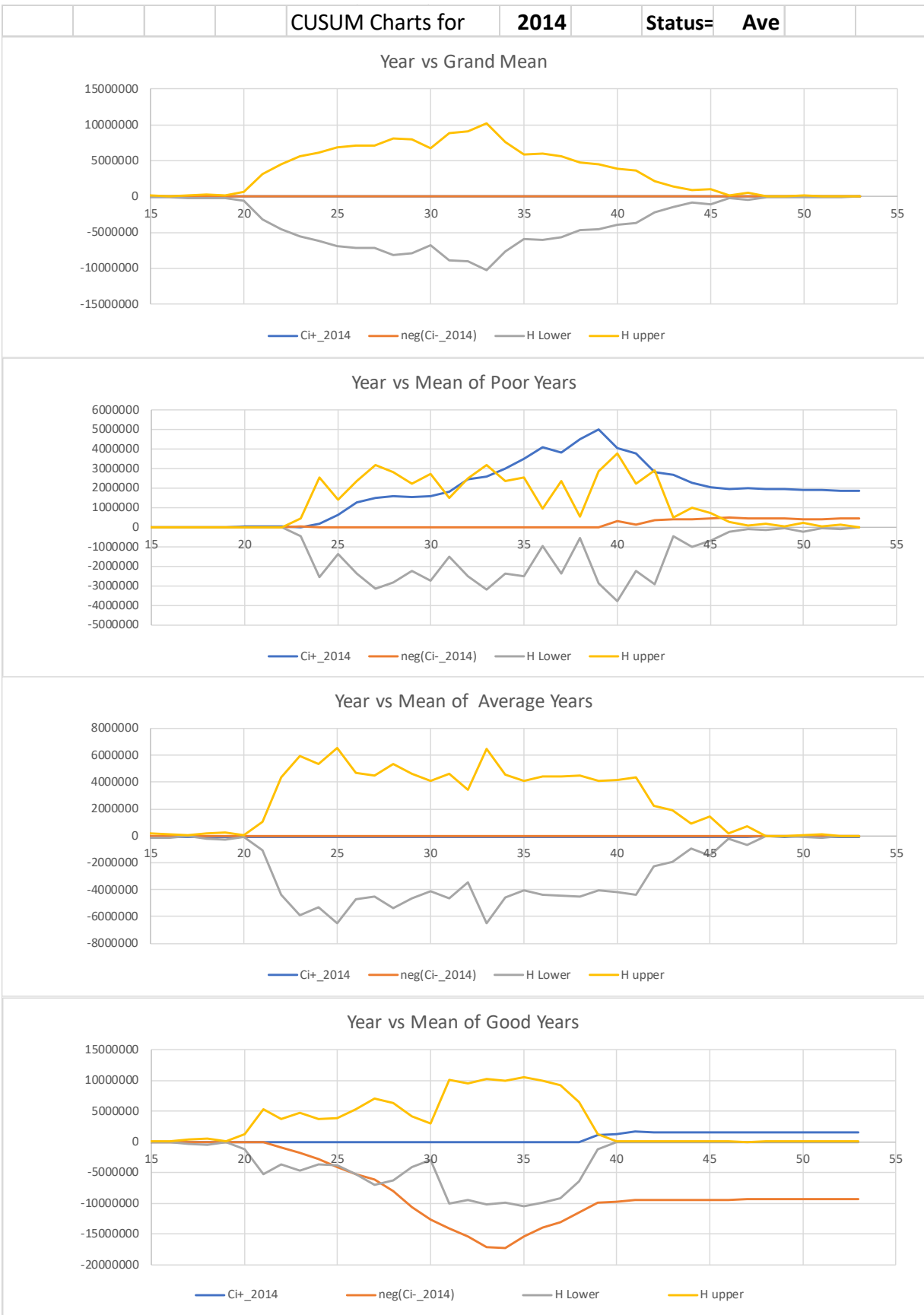


Figure 22. Cusum statistics for 2014 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2014 was classified a priori as an average year.



Figure 23. Cusum statistics for 2015 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2015 was classified a priori as a poor year.



Figure 24. Cusum statistics for 2016 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2016 was classified a priori as a poor year.

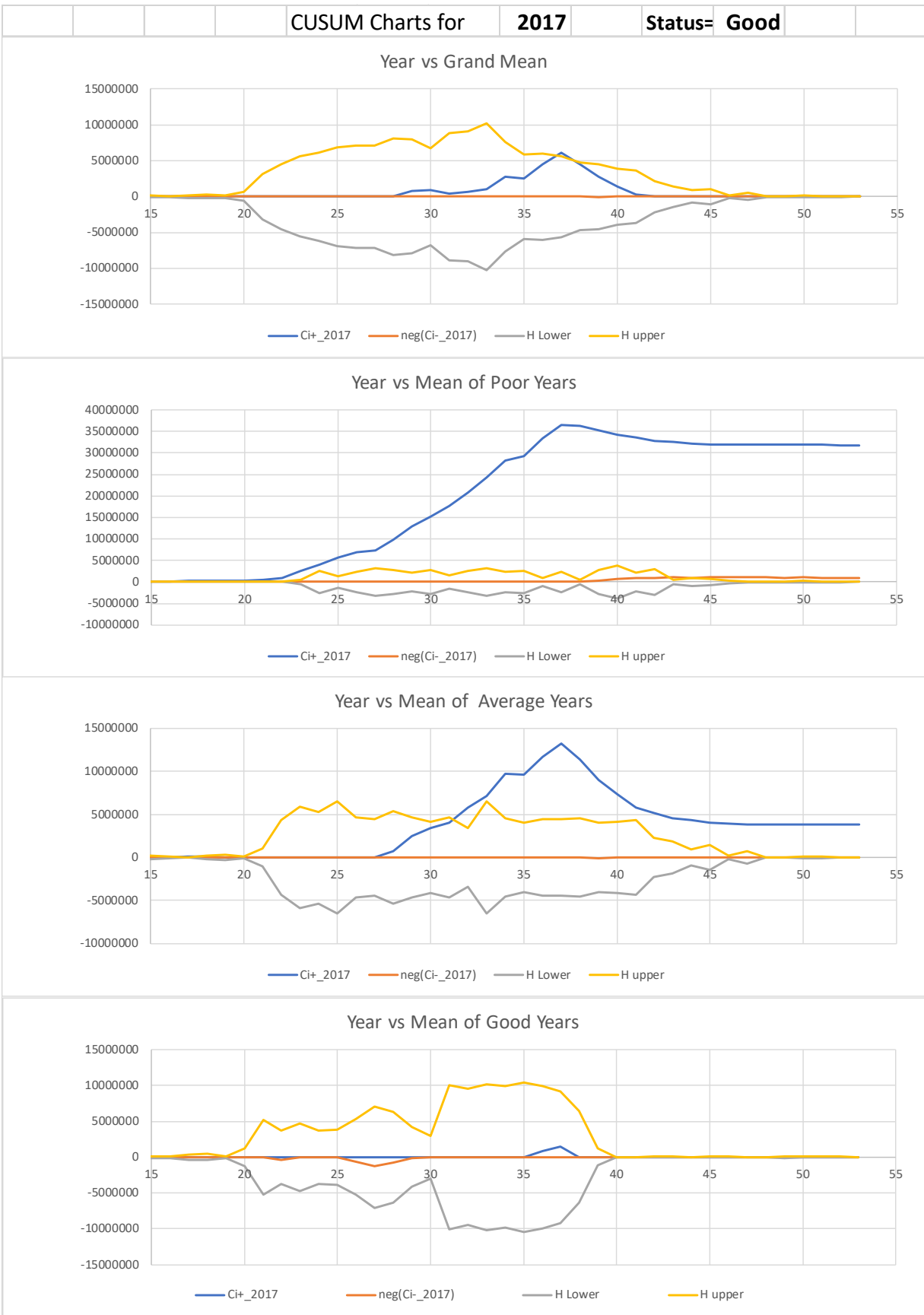


Figure 25. Cusum statistics for 2017 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2017 was classified a priori as a good year.



Figure 26. Cusum statistics for 2018 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2018 was classified a priori as a good year.



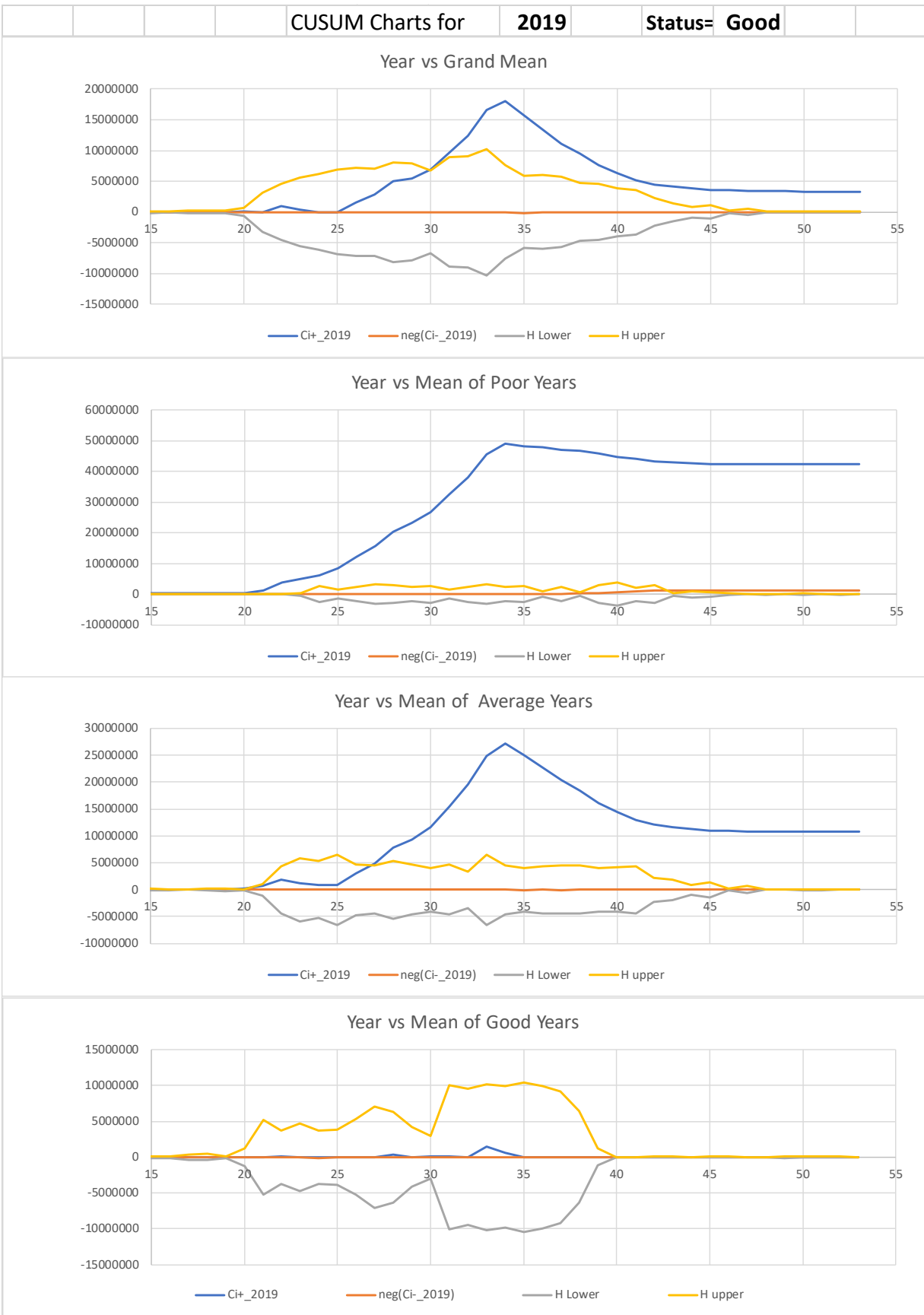


Figure 27. Cusum statistics for 2019 seasonal landings patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2019 was classified a priori as a good year.