

## **Part 2: Application of Cusum Method for In-Season Detection of Fishery Condition for Illex Squid; Mean Weight, 1997-2019**

Working Paper #16 Part 2 for Illex Working Group

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### **INTRODUCTION**

Illex squid grow rapidly and seasonal changes in their average weight in the fishery are important aspects of total landings. The average weight in the fishery, as measured by weekly biological samples, is a function of several factors. Growth of individuals and immigration of larger squid will increase the average size in the fishery. Similarly, cannibalism would tend to increase average size as the more vulnerable smaller individuals are eaten by larger squid. Conversely, slower than average growth, emigration of larger squid and recruitment of smaller squid would tend to diminish the average size of squid in the fishery. An overarching effect is the selectivity of the fishery which is a function of gear selectivity and the spatial pattern of fishing. For lack of a better term, the rate of change in average weight in the fishery can be called “apparent growth”. Disentangling these multiple factors affecting apparent growth is impossible at this point, but aging of squid samples is an important first step. The focus of this working paper is much simpler—can we use change in average weight as a real-time indicator of fishery status?

This paper builds on the methodology presented in Working Paper #16 Part 1 which focused on the use of a statistical quality control method (i.e., Cusum) to detect changes in weekly landings patterns. The emphasis herein is detection of apparent growth differences early in the fishing season. This allows a longer period for subsequent decision making and confirmation that the pattern detected was not a statistical aberration.

### **DATA**

Weekly average weights used in this analysis were graciously provided by Lisa Hendrickson. Through a joint effort of industry and the NEFSC (Hendrickson, Holmes and others), biological samples from freezer boats (SeaFreeze Ltd, Lapp per comm) were used to derive average weights by week for the same period of years, excluding 2007 and 2008. Summaries of squid average weights were provided by SeaFreeze Ltd and keypunched under the supervision of Lisa Hendrickson. These data constitute a valuable long term record of fishery conditions for Illex.

### **METHODS**

This paper applies the same Cusum methods described in Part 1 for total weekly landings. For completeness the methodology from Part 1 is repeated in the Appendix. In part 2 the response variable is average weekly mean weight of landed squid. The methodology requires some slight modifications for application to average weight.

Total landings were used to assign 3 categories of fishery system status. In the first category “Good” were annual totals that were more than one standard deviation above the long term mean. The “Poor” category was defined as total catches below one standard deviation and “Average” was defined as everything else (Figure 1). This method identifies the 5 years in which the quota was restrictive, and there was general agreement among fishermen about the

fishing conditions in the “poor” years. It is recognized that many factors can influence fishing success including market prices and availability of alternative species. As a first approximation this simple approach appears reasonable.

For comparative purposes, the landings-based classification was applied to average weight. Figure 2 illustrates the distribution of average weight samples for the entire year. Poor years (light green) tend have lower average weights compared to good years (dark green). Average years (red) have distributions similar to both poor and good years. This suggests that the overall average weight for a year might not be a good diagnostic tool. The seasonal changes, depicted as the ratio of the observed mean weight to the overall average weight can be visualized in Figure 3. (I thank Ben Galuardi, GARFO for providing a similar color-coded graphic to show seasonal changes in landings.) Here it is clear that seasonal patterns of higher apparent growth set up early, at least by week 29. Figure 4 illustrates the Lowess smoothed changes in median weights over the season for each status type. Good years tend to have higher median weight over the entire season. Poor years are not consistently distinguishable from average years until about week 28.

Let  $G_{w,y}$  equal the average weight observed in week  $w$  and year  $y$ . Essentially we would like to know what is the most likely seasonal distribution of catch rates the  $G_{w,y}$  is drawn from. Let  $\mu_{all,w}$ ,  $\mu_{good,w}$ ,  $\mu_{ave,w}$ , and  $\mu_{poor,w}$  represent the seasonal means for all samples, the good years, the average years and the poor years, respectively. For each week and type, it is possible to compute a standard deviation as well (i.e.,  $\sigma_{all,w}$ ,  $\sigma_{good,w}$ ,  $\sigma_{ave,w}$ , and  $\sigma_{poor,w}$  ) These values are used to create Figure 5. The Cusum statistics  $C_i^+$  and  $C_i^-$  can be redefined as

$$C_{w,y}^+ = \max [0, G_{w,y} - (\mu_{type,w} + K_{type,w}) + C_{w-1,y}^+ ]$$

$$C_{w,y}^- = \max [0, (\mu_{type,w} - K_{type,w}) - G_{w,y} + C_{w-1,y}^- ]$$

Where  $K_{type,w} = k \sigma_{type,w}$  where  $k$  is a constant=1.  $H$  is also redefined as  $H_{type,w} = h \sigma_{type,w}$  where  $h$  is a constant=5. Thus the Cusum process is specified to detect changes of 1 SD and is declared “out of control” when  $C_{w,y}^+ > H_{type,w}$  or when  $C_{w,y}^- < -H_{type,w}$ . In other words, if the Cusum statistics lie outside the  $H$  bounds, then one would reject the hypothesis that year in question was from a given type. Thus one would expect a putative good year, say 2018 to be rejected as an average or poor year by having a cusum statistic  $C_{w,2018}^+$  above  $H_{poor,w}$ , or  $H_{ave,w}$  for some week  $w$ . Similarly, a putative poor year should have  $C_{w,2016}^- < -H_{good,w}$  and  $C_{w,2016}^- < -H_{ave,w}$ . The test statistics  $C$  for 2018 and 2016 should be close the upper and lower  $H$  boundaries, respectively for the  $H_{all,w}$  comparison.

The variance estimates for average weight should be considered a first approximation. The samples are assumed to be representative of the vessel landings but is it not clear if the samples should be weighted by the proportion of total landings in a given size group. The degree to which individual samples may be under or over represented in the composite is unknown. If samples are roughly proportional to the landings by market category, then the existing sampling scheme is adequate. If not, then the variance estimate may be biased. Given these considerations, I reduced the buffer to  $K_{type,w} = 0.25 \sigma_{type,w}$  and the upper and lower decision bounds to  $H_{type,w} = 3 \sigma_{type,w}$ . Given an average standard deviation of over all years of ~35 g this

modification allows for detection changes apparent growth changes greater than about 8 g with decision bound on the order of 105 g.

## RESULTS and DISCUSSION

Seasonal landings data for each year were compared to the seasonal means and standard deviations derived from using all the data, the good years only, the average years only, and the poor years only (Figures 6 to 26). The visualizations are important because they allow for finer scale analyses of pattern. For example, in some years the **C** statistics exceeded the upper out of bounds limits for one week only and then returned to a lower value. In other instances, the **C** statistic closely tracks the upper or lower bound.

Table 1 provides a concise summary of the statistical results. Results can be scored by comparing the a priori designation of system state to the Cusum determination. Recall that these determinations were based on standardized average landings (Fig. 1) rather than the apparent growth patterns. Those years classified as good years {1998, 2004, 2017, 2018, 2019} were exceeded the upper H limit for the poor years {2001,2002, 2003, 2013, 2015, 2016} by week 28. The median detection time was 22 weeks. This suggests that early detection of a “better than poor” year is possible about mid July. The decision time to reject the “average” year {1997, 1999, 2000, 2005-2007, 2010-2012, 2014} was evident between week 27 and 35 for 2004, and 2017-2019, but it was not detectable for 1998. Poor years {2001,2002, 2003, 2013, 2015, 2016} were significantly different from good years (i.e., less than the lower H bound) by weeks 28 to 32. In terms of decision making, good years could be identified as better than poor in 5 of the 5 years and better than poor in 4 of 5 years. In other words, good years could be identified as “good” in 80% of the cases. Poor years could be excluded from consideration as a good year in 5 of 5 instances.

In the set of years pre-classified as average {1997, 1999, 2000, 2005-2007, 2010-2012, 2014} the decision making becomes more problematic. To simplify matters, assume that a decision later than say week 36 (~end of September). Apparent growth in the 2000, 2005,2010-2012, and 2014 average years were found to be below the good year growth trajectory by week 27 to 31. The average years 1997 and 2006 actually exceeded the “good” year trajectory but this was not determined until week 39 and 40 respectively. Apparent growth trajectories in 1999 and 2009 were not different from the good year trajectory.

In an ideal system, one wants to detect the condition {good, average, bad} as early as possible, leaving more time for making a decision and implementing the regulatory process. If the decision-making problem is recast as simply identifying the “good” year at a time when a management decision could be made (week 36), then one would conclude that the Cusum method works 80% of the time. There were no false positives, i.e., years classified as poor or average years exceeding the average year trajectory before week 37. In instances where the test year could not be distinguished from the average or good year (2000, 2009) the default decision would be to call the year as not significantly different from an average year.

All of the above discussion assumes that the initial classification of year status is correct. Therefore, this is one of the most important considerations in the application of this method for real time identification. Solicitation of input from harvesters and processors is essential for this determination. Other candidate metrics for system identification could also be included (See Working Paper #13 for some exploration of this concept.) Other potential improvements to the application of Cusum to real-time management are discussed in Working Paper #16, part 1)

## ACKNOWLEDGEMENTS

This working paper would not have been possible without the foresight to identify and collect the biological data on average weights in the fishery on a weekly basis. I thank Lisa Hendrickson for that foresight and representatives of SeaFreeze Ltd (especially Meghan Lapp, Glenn and Kyle Goodwin) for their contributions of data for the over 20 years. I also thank Jason Didden for his leadership and encouragement on this project.

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Table 1. Summary of Cusum performance for detecting system state (good, average, poor) using slack variable  $K=0.25$  SD and control bounds  $H=\pm 3SD$  limits. The response variable is average weight per week. Entries represent the week when the Cusum first exceeded the control limit. The sign following the number represents whether the Cusum statistics exceeded the upper bound (+) or fell below the lower bound (-). Data for 2007 and 2008 were not available when this report was prepared.

Year	Classification	First Out of Bounds Detection Year			
		All Years	Poor Years	Average Years	Good Years
1997	Ave	35+	29+	37+	39+
1998	Good	34+	22+	none	none
1999	Ave	none	29+	none	none
2000	Ave	32-	none	none	31-
2001	Poor	33-	none	34-	28-
2002	Poor	40-	35+	none	32-
2003	Poor	none	37+	none	32-
2004	Good	23+	20+	33+	none
2005	Ave	37-	34+	38-	30-
2006	Ave	34+	30+	36+	40+
2007					
2008					
2009	Ave	41+	29+	none	none
2010	Ave	none	29+	none	33-
2011	Ave	38-	none	38-	32-
2012	Ave	38-	none	38-	33-
2013	Poor	29-	none	33-	28-
2014	Ave	29-	none	29-	27-
2016	Poor	33-	none	33-	32-
2017	Good	32+	28+	33+	none
2018	Good	26+	22+	27+	none
2019	Good	31+	22+	35+	none

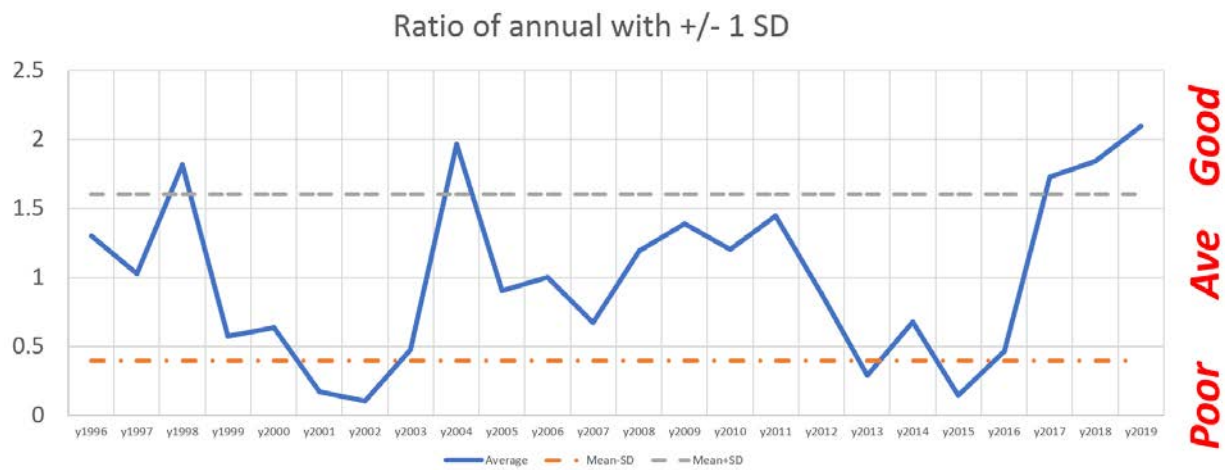


Figure 1. Designation of good, average and poor fishing years based on total landings. The dashed red lines represent +/- 1 SD of the mean. Annual catches were normalized by dividing observations by the overall mean.

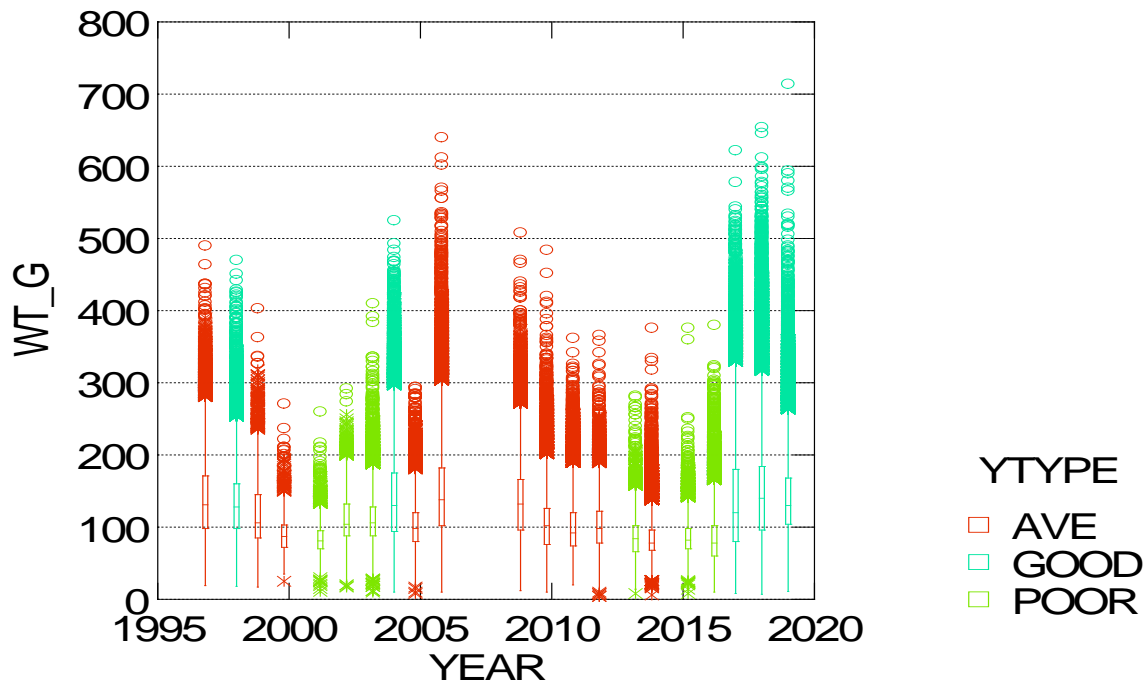


Figure 2. Box plots of average weights (g) of *Illex* squid obtained from biological samples obtained from SeaFreeze Ltd. Center line of box is the median, and the boundaries of the box represent the interquartile range. The “whiskers” on the boxes are 1.5 \* Interquartile Range. Squid larger than whisker limits were observed in all years. Data for 2007 and 2008 were not available when this report was prepared. Colors of bars are coded by type based on the analyses of seasonal landings patterns.

Type	Ave	Ave	Good	Ave	Ave	Poor	Poor	Poor	Good	Ave	Ave	Ave	Ave	Ave	Ave	Ave	Poor	Ave	Poor	Poor	Good	Good	Good		
Type	Fishing Year																								
Week	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	
21		0.49	0.50						0.83														0.80	0.69	
22			0.60						0.79														0.80	0.84	
23			0.69	0.58			0.68	0.70	0.85						0.44	0.61	0.59					0.66	0.88	0.52	
24		0.48	0.86	0.64		0.61	0.80	0.74	0.91	0.75					0.86	0.58	0.45	0.61				0.70	0.86		
25			0.85	0.73	0.81	0.71	0.74	0.82	0.84	0.67	0.68			0.62	0.72	0.74	0.58	0.60				0.82	1.01	1.04	
26		0.79	0.84	0.78			0.73	0.86	0.88	0.97	0.74	0.70		0.81	0.95	0.64	0.73	0.50	0.56	0.55	0.63		1.23	0.99	
27		0.96	0.92	0.85	0.85	0.72	0.79	0.96	0.99	0.73	0.90			0.87					0.56				0.98	1.37	0.93
28		0.73	1.08	0.84	0.86	0.75	0.82	0.96	1.34	0.77	0.93			0.91	0.97	0.64	0.86	0.52	0.62			1.13	1.51	1.05	
29		1.03	1.15	1.23	0.78	0.64	0.96	1.05	1.06	0.88	1.03			1.03	1.01	0.79	1.00			0.68	0.70	1.26	1.44	1.19	
30		1.24	1.22	1.11	0.66	0.54	0.92	0.79	1.18	0.91	1.03			1.23	1.04			0.69	0.60			1.54	1.61	1.25	
31		1.21	1.29	1.10	0.61		0.95	0.80	1.33	0.95	1.33			1.08		0.82			0.66	0.63	0.64	1.73	1.34		
32		1.16	1.28	1.06	0.66		0.85	0.97	1.45	1.01	1.49			1.34	1.04		0.81	0.74	0.75			1.70	1.66	1.43	
33		1.38	1.34	0.98			0.95	0.81	1.48	0.98	1.35			1.46		1.06		0.75	0.67		0.54	1.66	1.89	1.46	
34		1.34	1.49	1.01		0.73	0.92		1.64	0.98	1.53			1.21	1.01	0.95	0.95		0.80	0.73	0.63	1.75		1.73	
35		1.42	1.46	1.15		0.67	0.92	0.88	1.35	0.83	1.60			1.46	1.01	0.88	0.85	0.73	0.77		0.58	2.05			
36		1.47		0.98		0.71	0.71	0.96	1.06	0.80	1.39			1.32	1.02		0.77		0.90	0.69	0.64	1.92		1.64	
37		1.60		1.24				0.86	1.08	0.89	1.72			1.23			0.96	0.96	0.75		0.74	1.95		1.90	
38		1.33		1.24	1.19		0.98	1.19	1.20	0.93	1.78						0.86		1.01	0.67	0.77				
39		1.47		1.44	1.16			0.99		0.80	1.53			1.13			0.96	0.86	0.81		0.98	0.72		1.69	
40		1.46		1.33				1.02		0.84	1.38						0.97	0.92	0.88						
41		1.16					0.95	1.01						1.28			0.99	0.36	0.81						
42		1.59					0.89				1.47						0.95	0.92	0.66		0.66	0.87	1.57		
43		1.47					0.92		1.17		1.27			1.13								0.85		0.56	
44		1.22						1.22								1.18		1.04				0.99		2.32	

Figure 3. Seasonal pattern of average weight in fishery for 1997-2019. Weekly means are standardized by dividing each observation by overall mean for the entire time series (= 123 g). Red colors denote higher values, blue denote lower values.

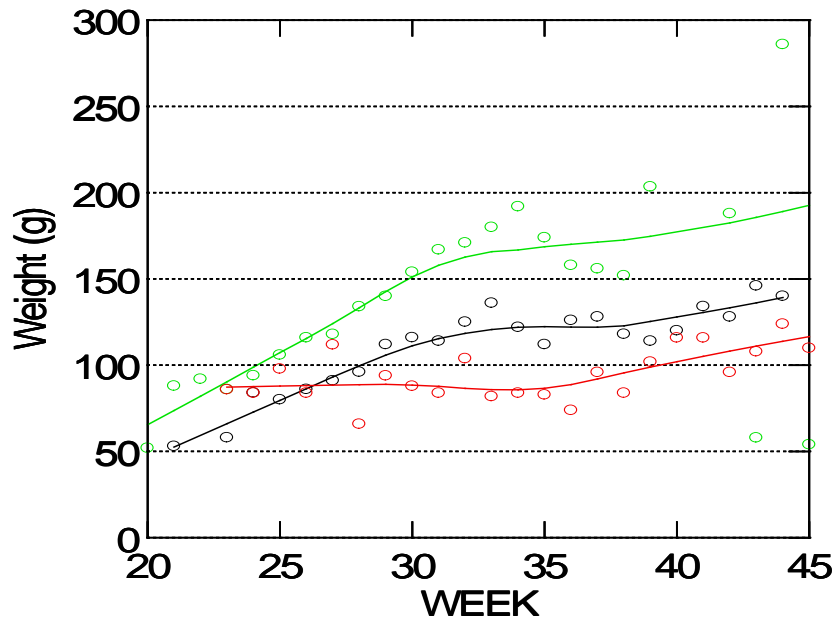


Figure 4. Lowess smooths of median weights (g) of sampled *Illex* squid by fishery system state (poor, average, good). Biological samples obtained from SeaFreeze Ltd. Data are pooled over years. Colors of symbols are coded by system state based on the analyses of seasonal landings patterns. (red=poor, black=average, green=good).

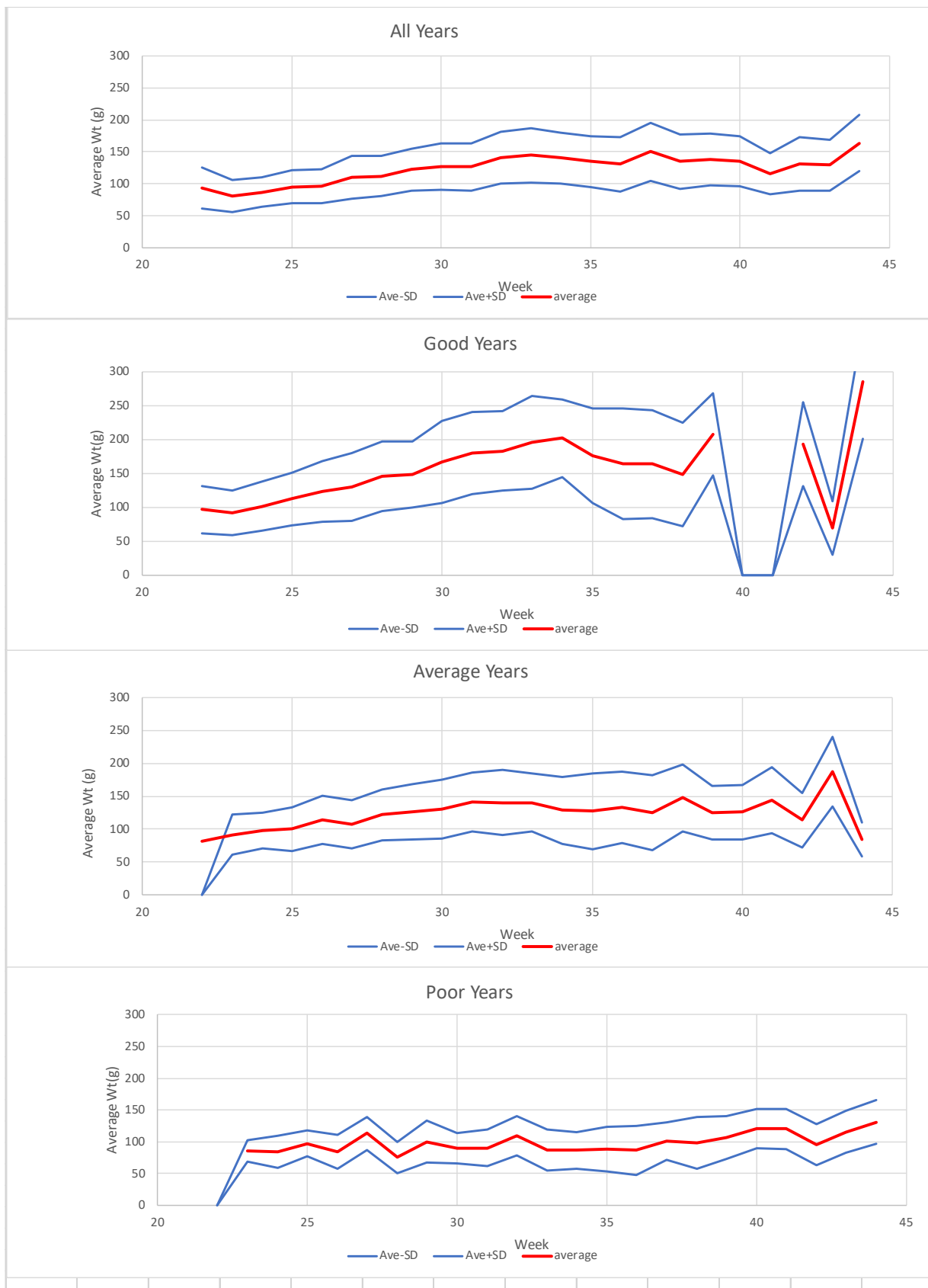


Figure 5 Seasonal average weight patterns for all years (top row), good years (row 2), average years (row 3) and poor years (bottom row) expressed in terms of Average weight (g) per week. Red line is the overall mean, blue lines are +/- 1 SD of mean. Biological samples were not available for Weeks 40-42 in good years owing to fishery closure.



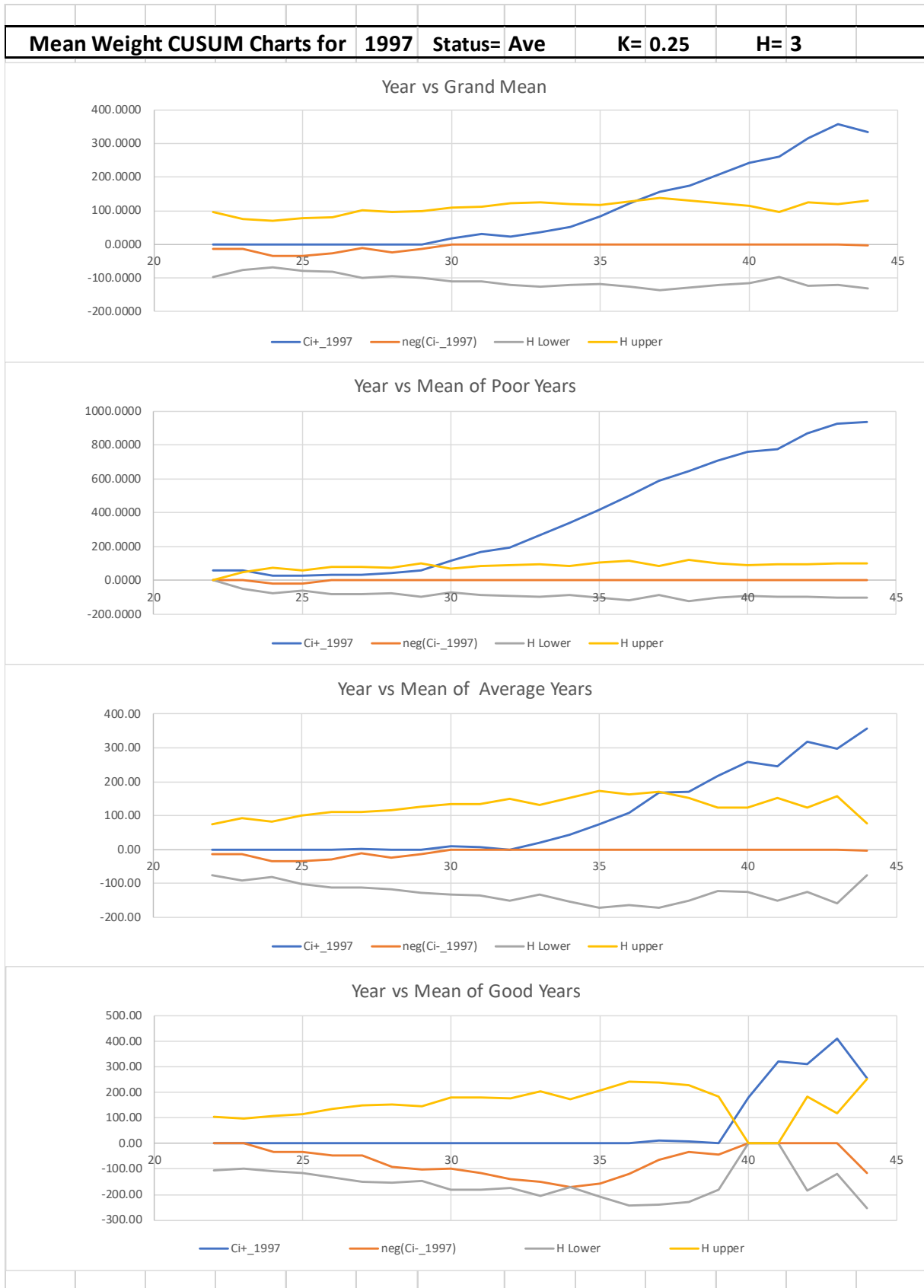


Figure 6 Cusum statistics for 1997 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 1997 was classified a priori as an average year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$

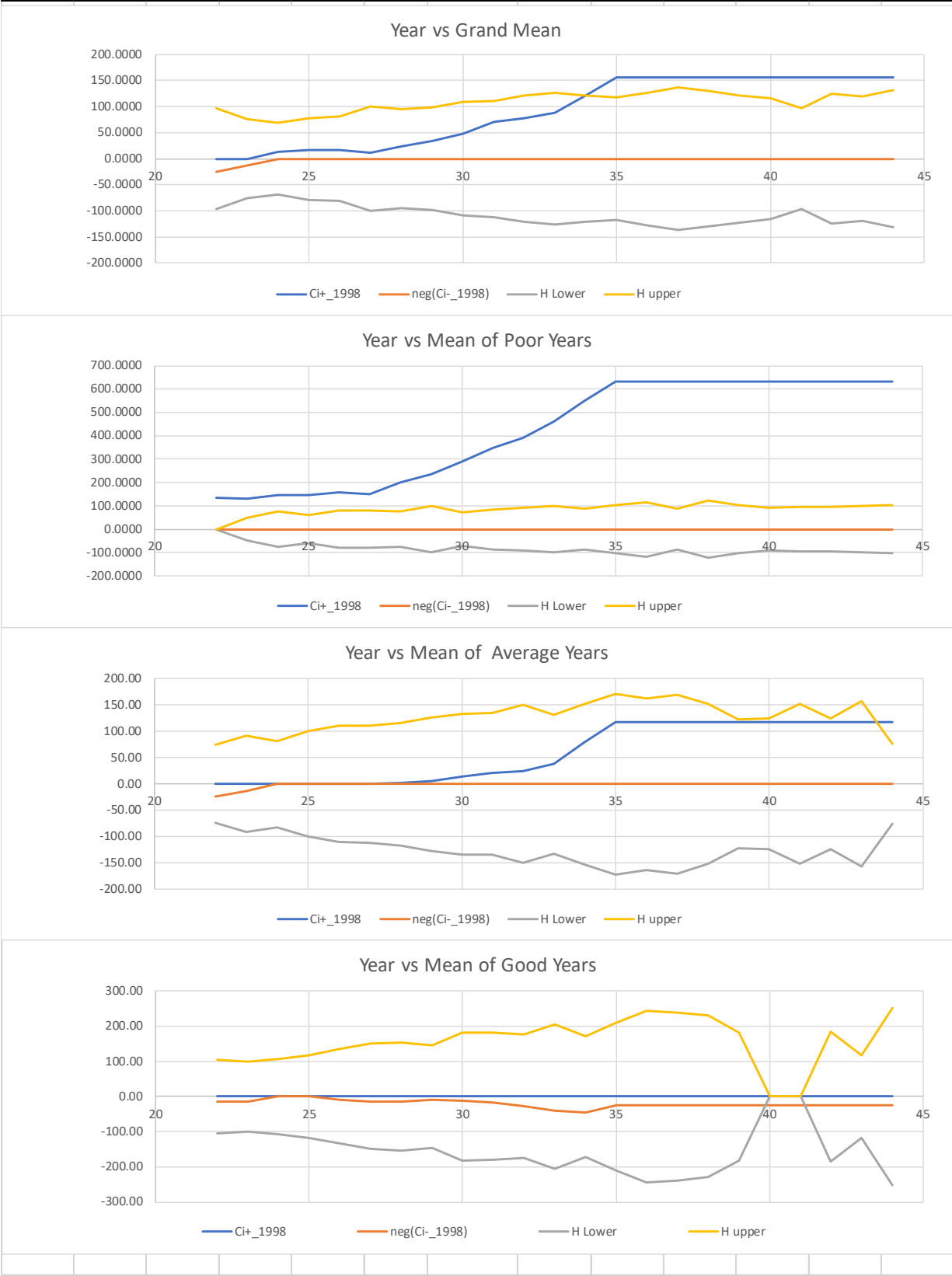


Figure 7. Cusum statistics for 1998 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 1998 was classified a priori as a good year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$

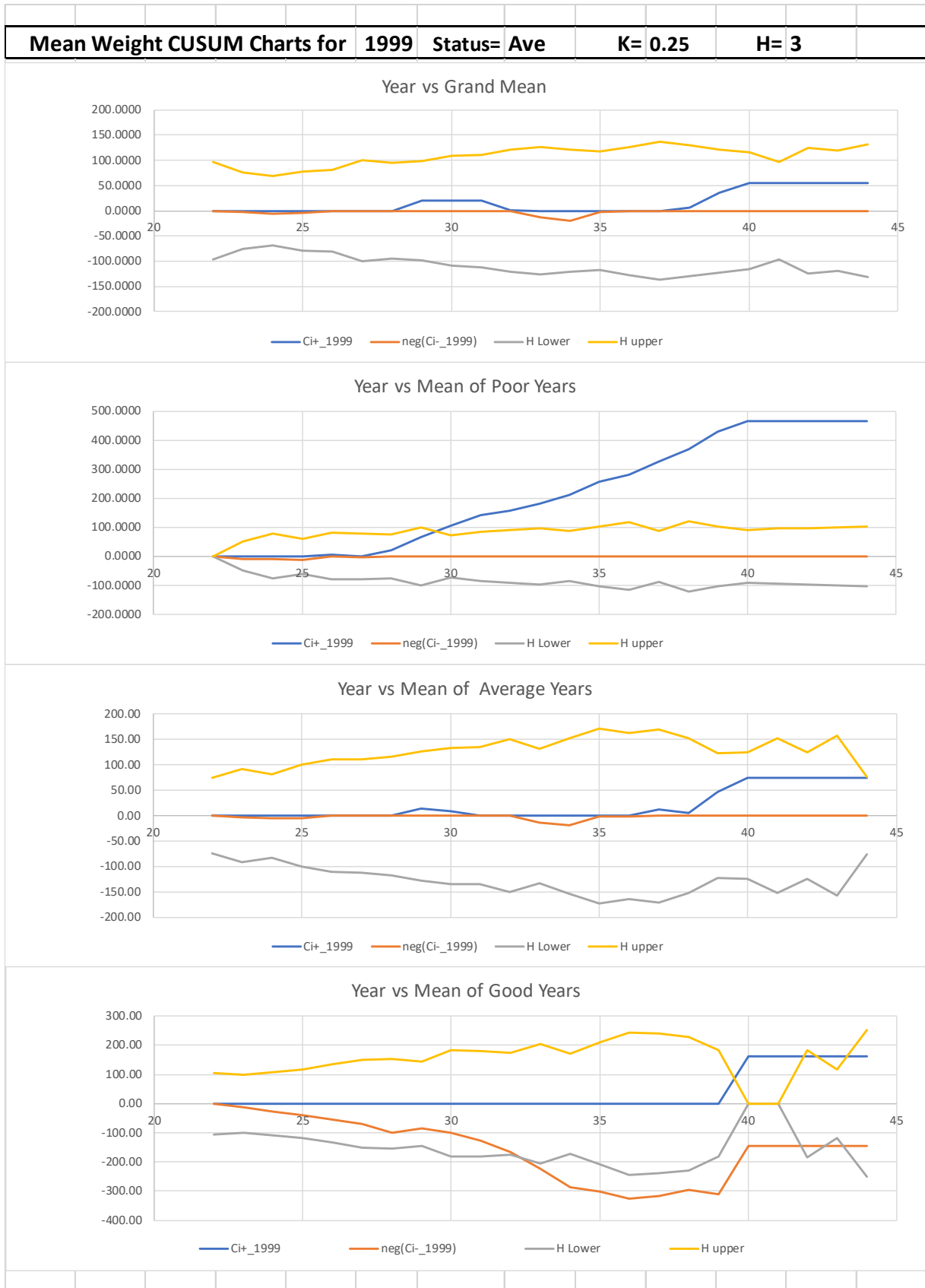


Figure 8. Cusum statistics for 1999 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 1999 was classified a priori as an average year.  $K=0.25\sigma$ ,  $H=+/- 3\sigma$



Figure 9. Cusum statistics for 2000 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2000 was classified a priori as an average year.  $K=0.25\sigma$ ,  $H=+/- 3\sigma$



Figure 10. Cusum statistics for 2001 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2001 was classified a priori as a poor year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$

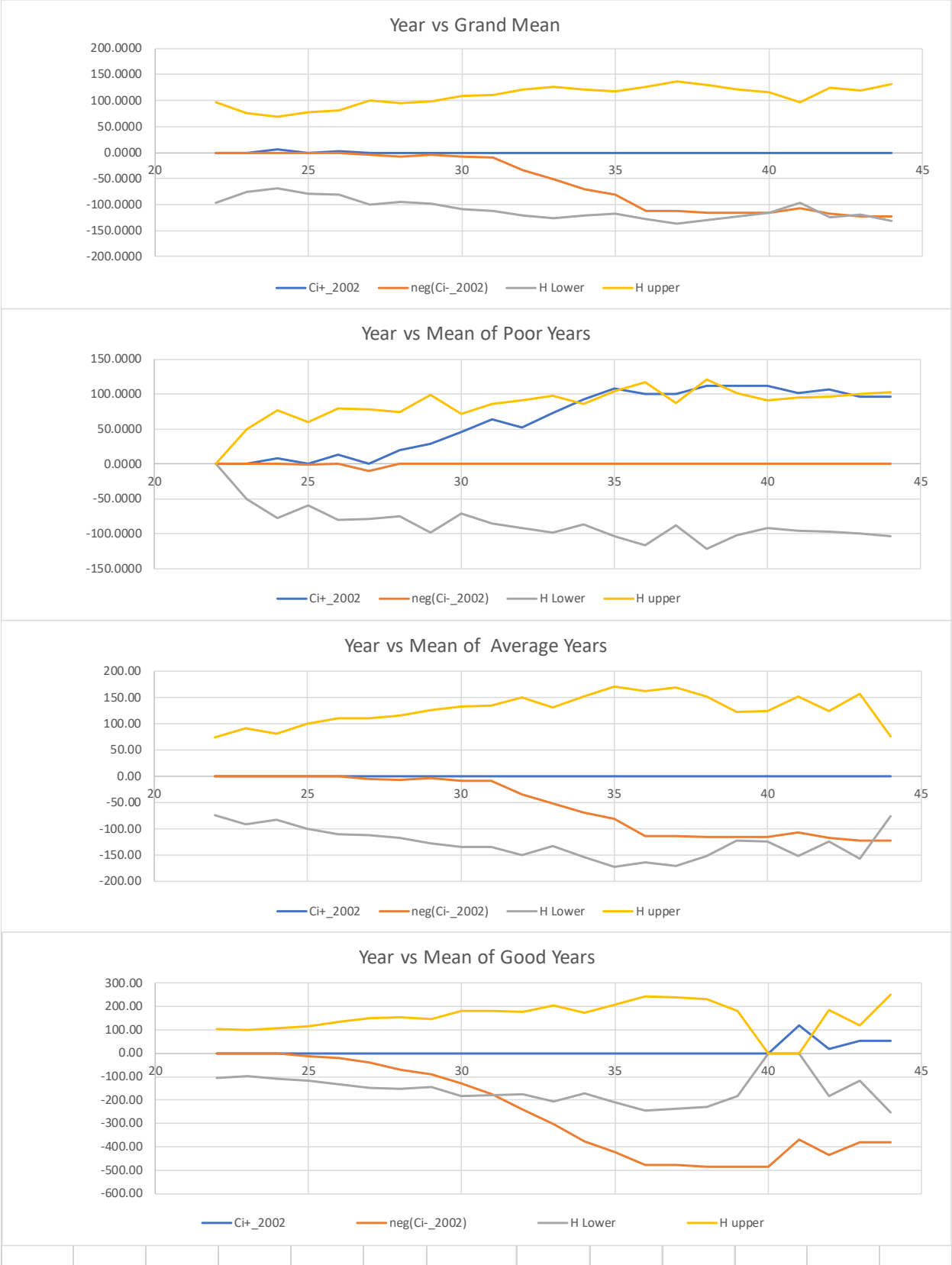


Figure 11. Cusum statistics for 2002 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2002 was classified a priori as a poor year.  $K=0.25\sigma$ ,  $H=+/- 3\sigma$

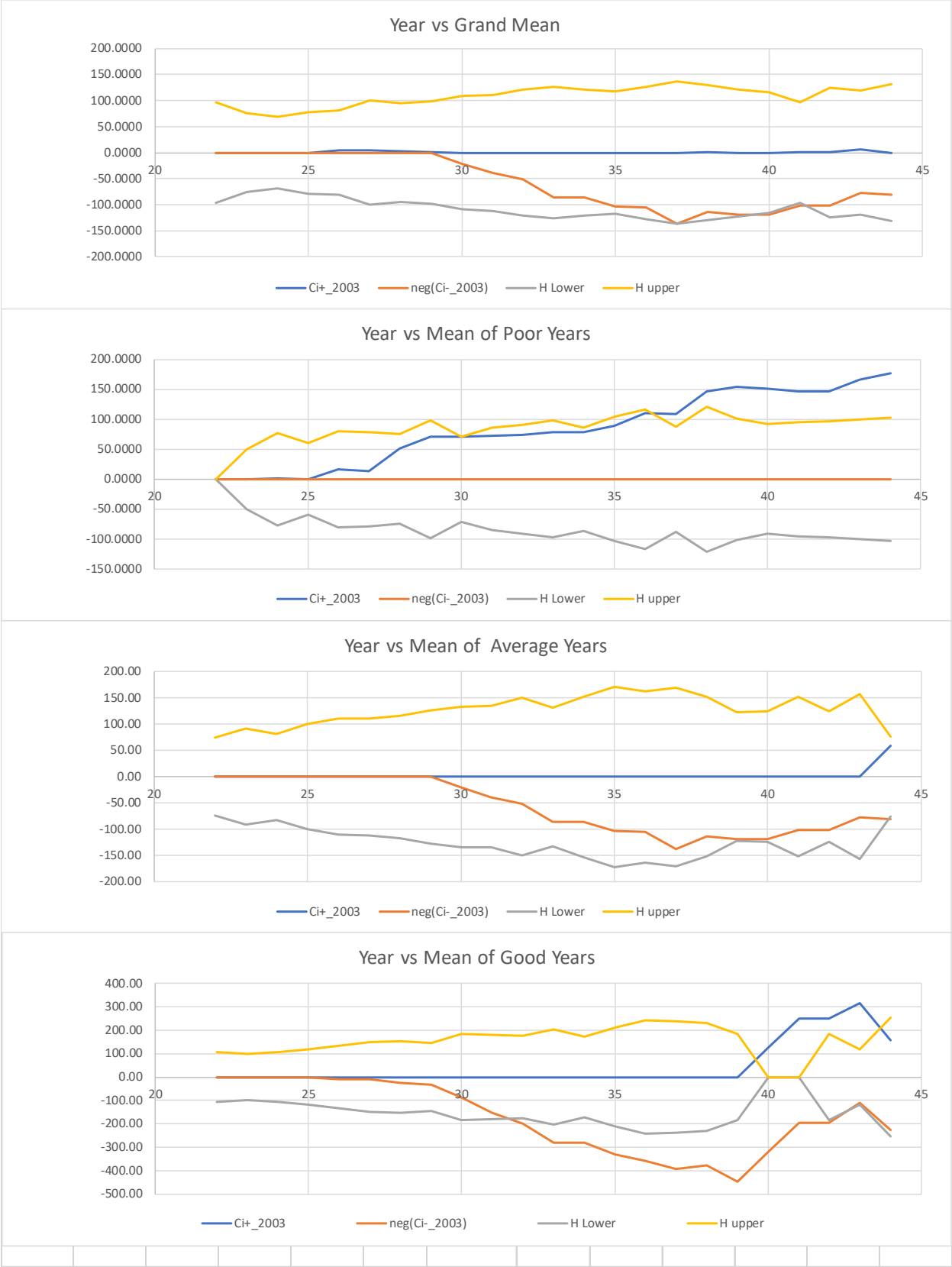


Figure 12. Cusum statistics for 2003 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2003 was classified a priori as a poor year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$

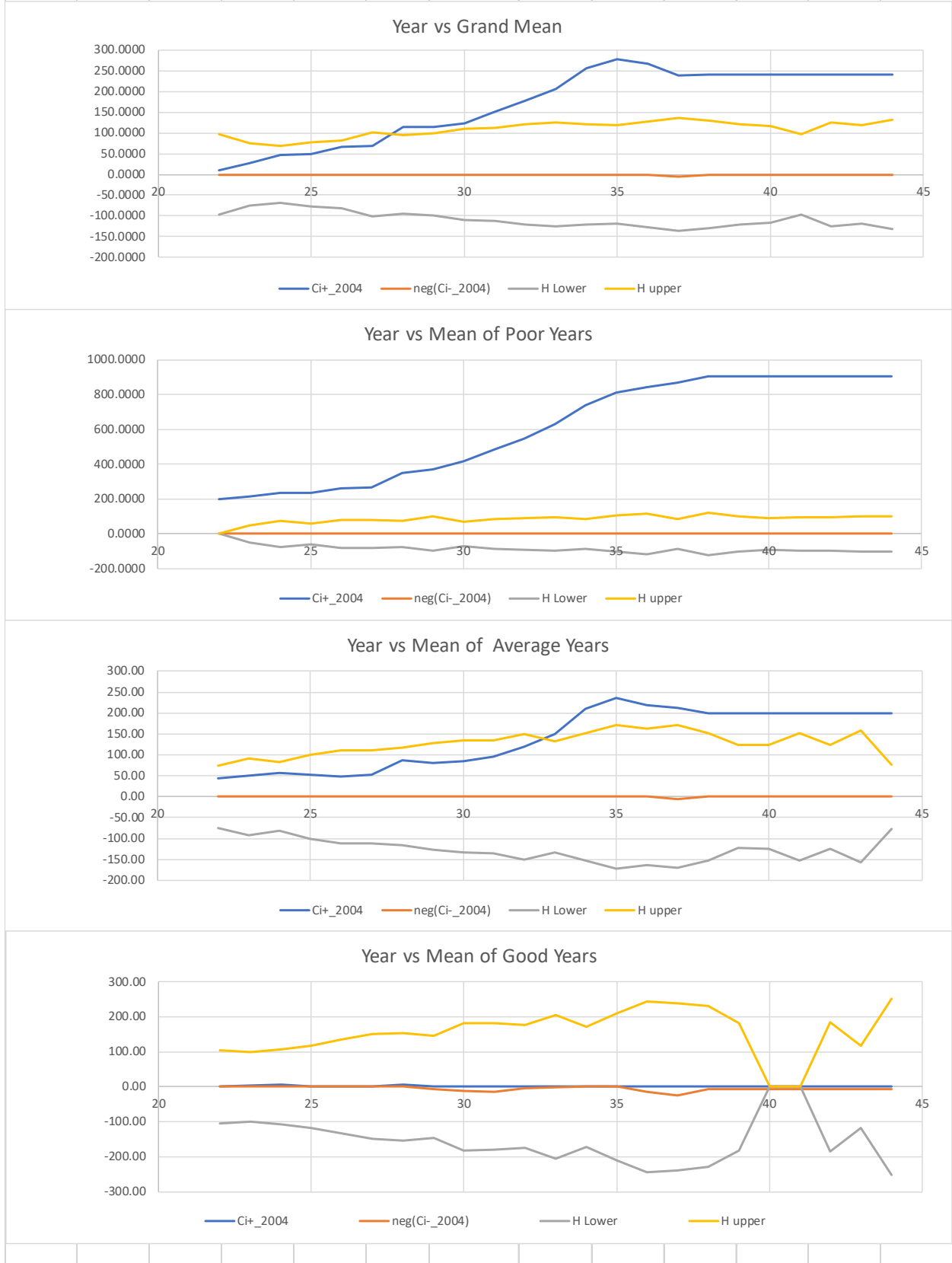


Figure 13. Cusum statistics for 2004 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2004 was classified a priori as a good year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$





Figure 14. Cusum statistics for 2005 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2005 was classified a priori as an average year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$



Figure 15. Cusum statistics for 2006 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2006 was classified a priori as an average year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$



Figure 16. Cusum statistics for 2009 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2009 was classified a priori as an average year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$

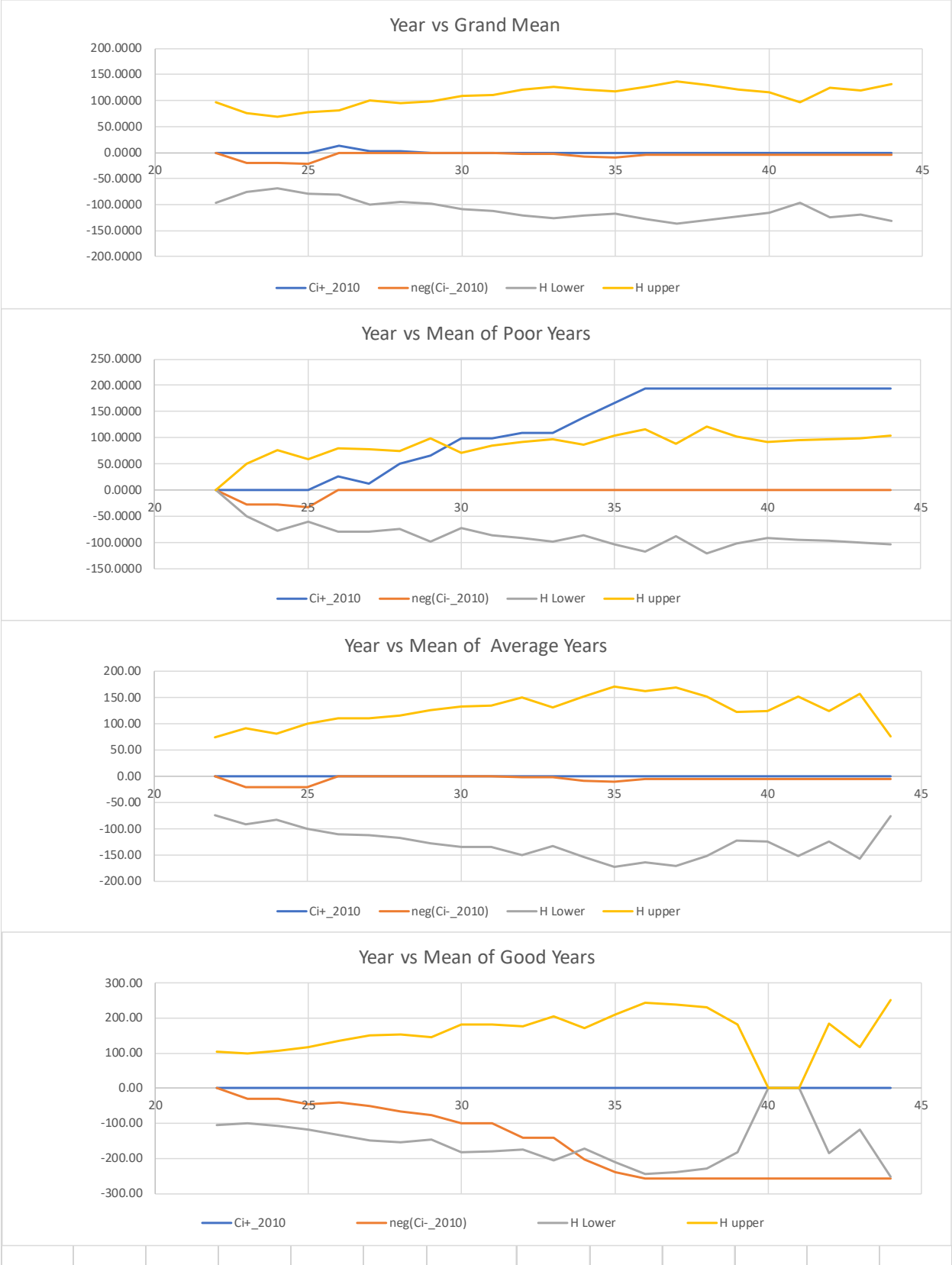


Figure 17. Cusum statistics for 2010 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2010 was classified a priori as an average year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$



Figure 18. Cusum statistics for 2011 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2011 was classified a priori as an average year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$



Figure 19. Cusum statistics for 2012 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2012 was classified a priori as an average year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$

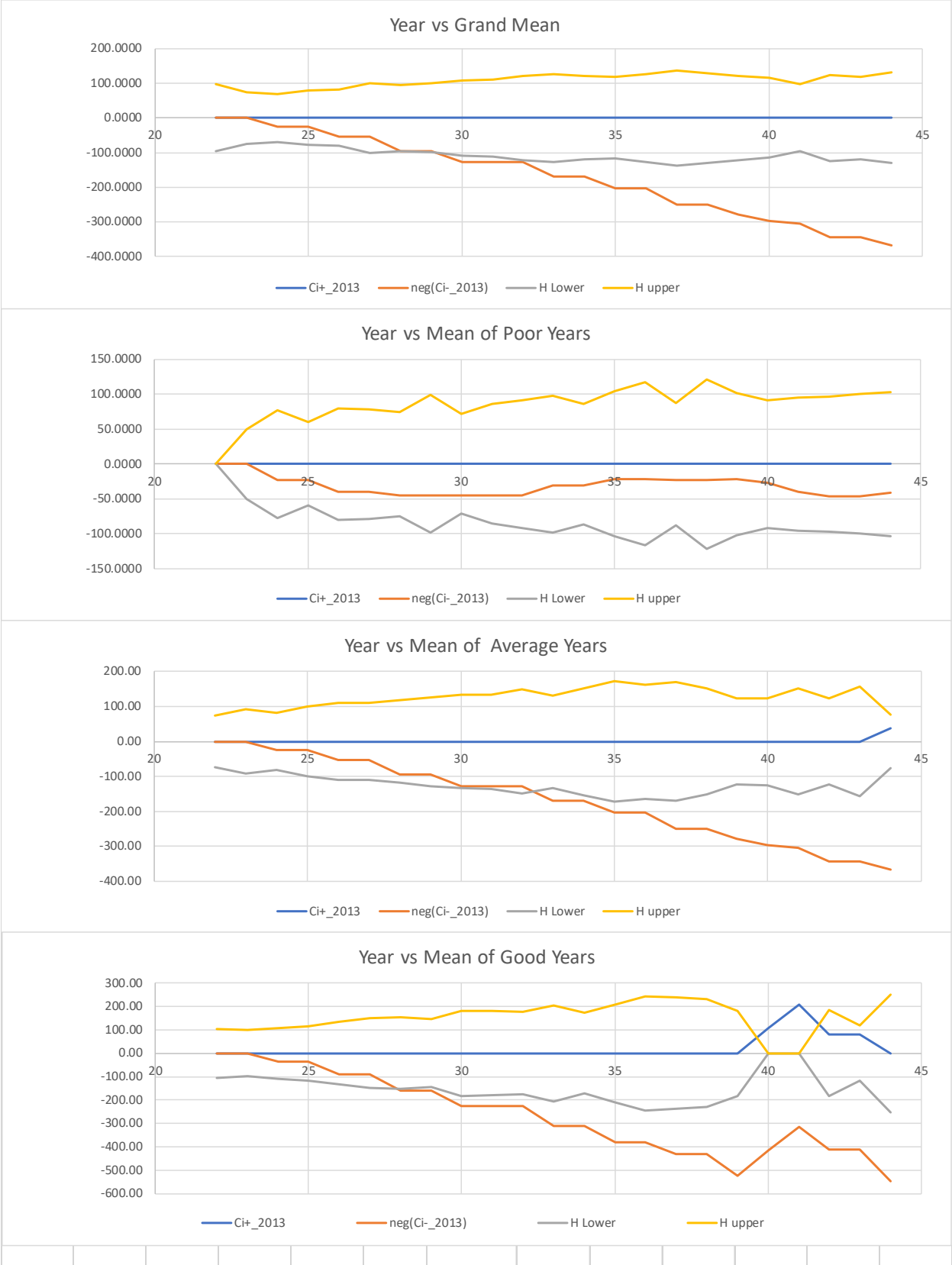


Figure 20. Cusum statistics for 2013 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2013 was classified a priori as poor year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$



Figure 21. Cusum statistics for 2014 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2014 was classified a priori as an average year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$





Figure 22. Cusum statistics for 2015 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2015 was classified a priori as poor year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$



Figure 23. Cusum statistics for 2016 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2016 was classified a priori as poor year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$

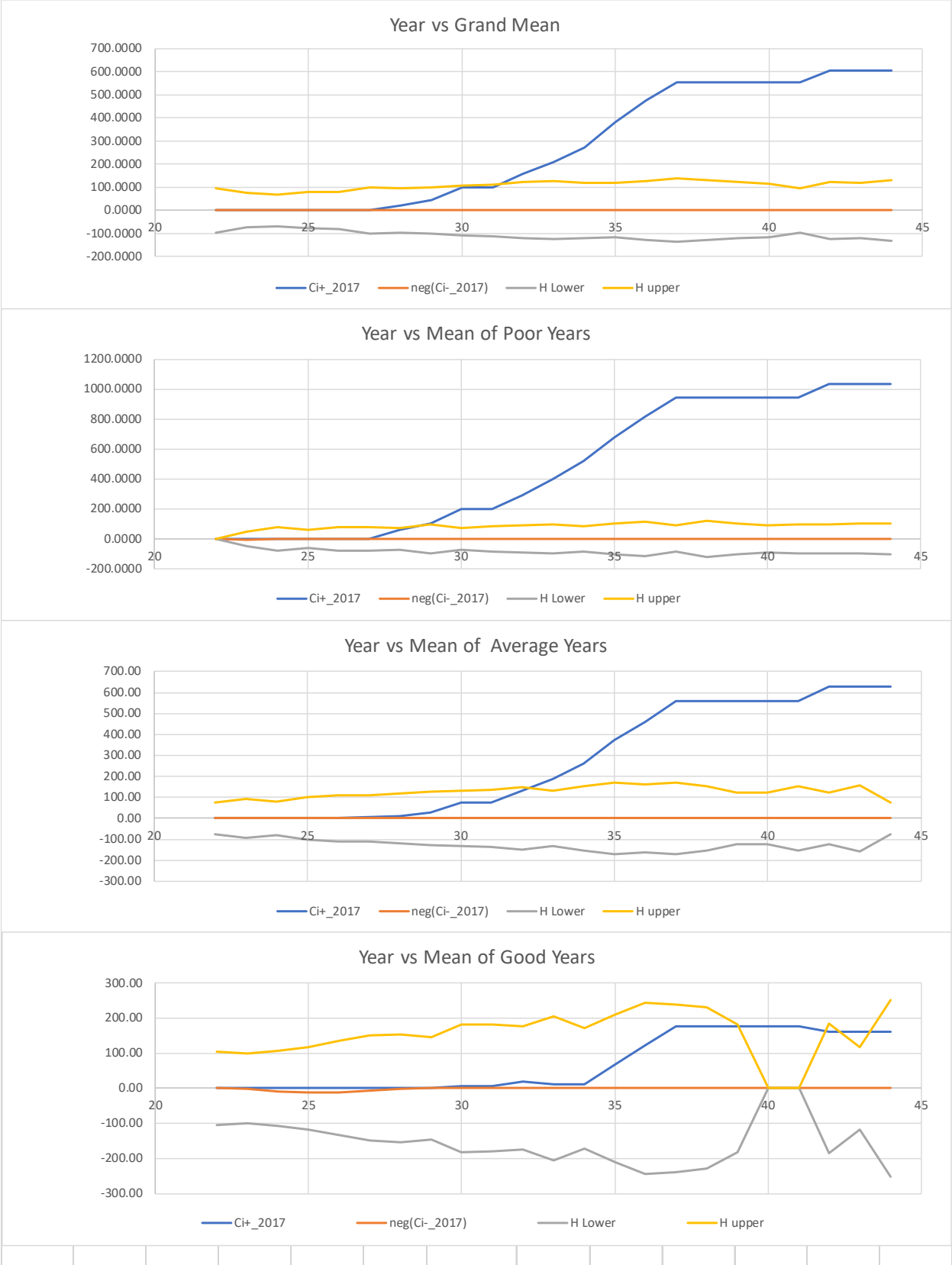


Figure 24. Cusum statistics for 2017 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2017 was classified a priori as good year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$



Figure 25. Cusum statistics for 2018 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2018 was classified a priori as good year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$

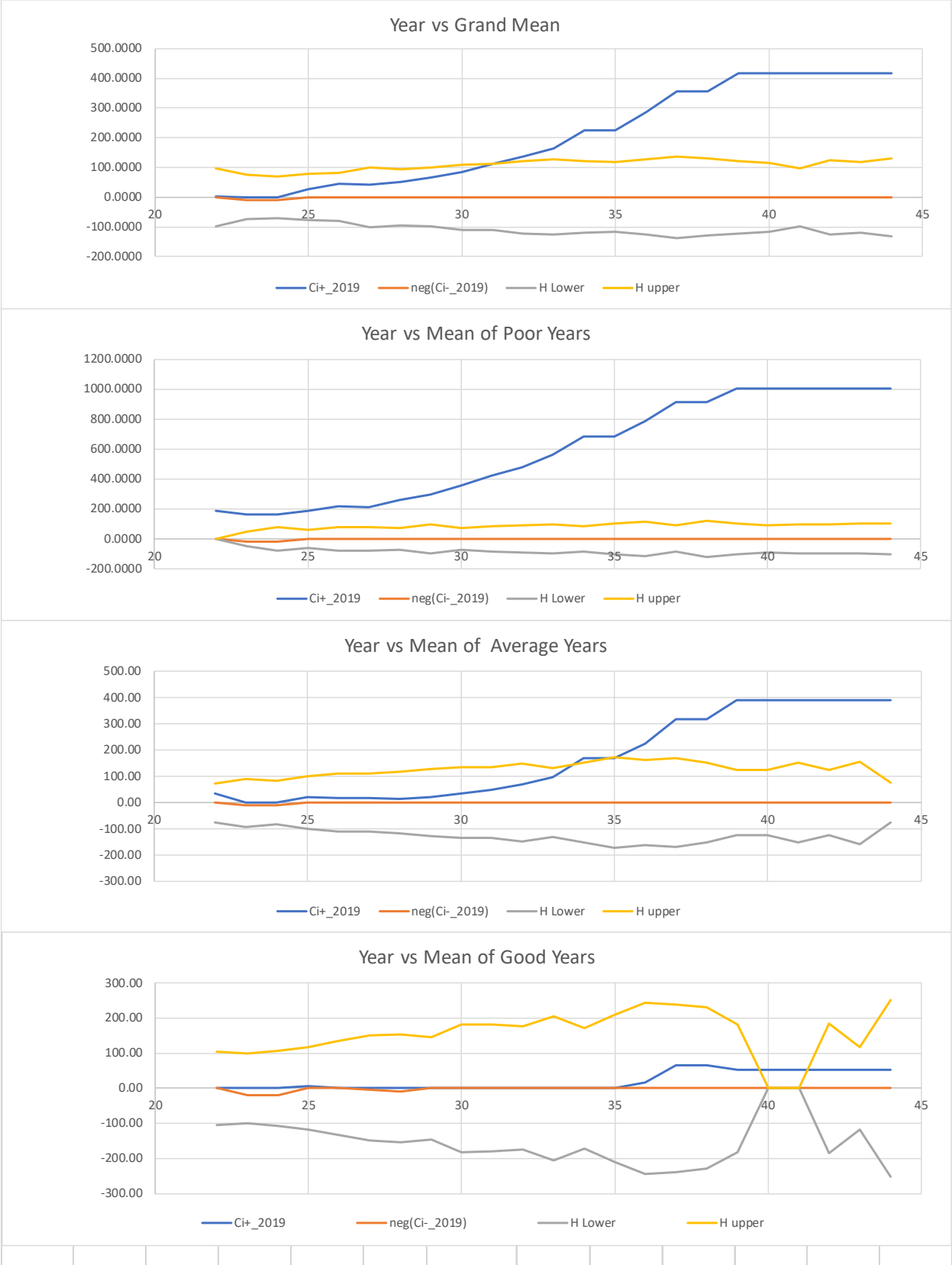


Figure 26. Cusum statistics for 2019 seasonal average weight patterns to season patterns based on all the data (top row), poor years (row 2), average years (row 3) and good years (bottom row). Orange line represents the upper H boundary, gray line the lower H boundary. C+ is shown in blue; C- is shown in red. Based on landings, 2019 was classified a priori as good year.  $K=0.25\sigma$ ,  $H=\pm 3\sigma$

**APPENDIX: General description of Cusum method (from Part 1 of this report)**

Cumulative sum (or Cusum) methods are widely used in statistical process control where real-time decision-making is important for maintaining quality standards for a product or a process. The basic idea is to collect information that appropriately identifies when the process is out of control and avoids erroneous identification of out-of-control processes. Letting a process continue when the quality has degraded is bad for future revenue (i.e., defective widgets). Conversely, incorrectly stopping a production process to fix something that is working fine, increases costs and reduces profits.

The Cusum method was first proposed by Page (1954, 1961) and has been refined by many authors since then. The basic concept is simple. Consider a sequence of random variables  $\mathbf{x}_i$  drawn from a normal distribution with mean  $\mu_0$  and variance  $\sigma^2$ . If  $\mu_0$  is known, then the expected value of  $\Sigma(\mathbf{x}_i - \mu_0) = 0$ . If the mean changes to a new value  $\mu_n$ , but  $\mu_0$  is assumed, then  $\Sigma(\mathbf{x}_i - \mu_0)$  will increase with the number observations. Conversely, if the new mean is less than the hypothesized mean, then the cumulative sum will become increasingly negative. Cusum methods take advantage of this expected change to define upper and lower control limits sufficient to detect such changes in real time. Among the advantages of Cusum is earlier detection compared to other methods such as Shewhart control charts.

The methodology herein is based on the so-called tabular method of Montgomery (1996) but modified to account for a seasonally varying mean and variance.

First let's consider a constant mean and variance. The upper and lower cusums are denoted as  $C_i^+$  and  $C_i^-$  and defined by the following recursive equations:

$$C_i^+ = \max [0, x_i - (\mu_0 + K) + C_{i-1}^+ ]$$

$$C_i^- = \max [0, (\mu_0 - K) - x_i + C_{i-1}^- ]$$

Starting values for  $C_i^+$  and  $C_i^-$  are both set to zero for  $i=1$ . The parameter  $\mathbf{K}$  is called the "slack" variable as it acts like a buffer or tolerance level. Changes in  $\mathbf{x}_i$  of less than  $\mathbf{K}$  are essentially zeroed out. For example  $C_{i+1}^+ > C_i^+$  increases only when  $\mathbf{x}_i > \mu_0 + \mathbf{K}$  and  $C_{i+1}^- < C_i^-$  only when  $\mathbf{x}_i < \mu_0 - \mathbf{K}$ .

$\mathbf{K}$  is generally expressed as a function of the standard deviation. The parameter  $\mathbf{K}$ , written as  $\mathbf{K} = \delta\sigma$  represents the magnitude of the change one wishes to detect in  $\mu$ . The expected number of samples necessary to detect this change is called the average run length (ARL). In other words, ARL is the average number of samples required before determination of true out of control state is determined given an acceptable level of "slack" in the underlying process.

The ARL statistic is determined as a function of  $\mathbf{K}$  and another parameter called  $\mathbf{H}$  where  $\mathbf{H} = \gamma\sigma$ . Montgomery recommends letting  $\gamma$  equal 4 or 5 in order to detect changes within reasonably small number of samples (ARL). For  $\delta=1$  and  $\gamma=5$  the ARL is 10.4; if  $\gamma=4$  then ARL is 8.38. In general terms ARL decreases as  $\delta$  and  $\gamma$  increase. The process is judged to be out of control when  $C_i^+ > \mathbf{H}$  or  $C_i^- < -\mathbf{H}$ .

The above description of Cusum assumes that the mean  $\mu$  and variance  $\sigma^2$  are static over time. For our application we are interested in determining whether we can detect a change in an underlying mean that changes seasonally.

